Beginning and Intermediate Algebra

Chapter 7: Rational Expressions

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Chapter 7: Rational Expressions

7.1

Rational Expressions - Reducing

Rational expressions are expressions written as a quotient of polynomials. Example of rational expressions include:

$$\frac{x^2-x-12}{x^2-9+20} \quad \text{and} \quad \frac{3}{x-2} \quad \text{and} \quad \frac{a-b}{b-a} \quad \text{and} \quad \frac{3}{2}$$

As rational expressions are a special type of fraction, it is important to remember with fractions we cannot have zero in the denominator of a fraction. For this reason, rational expressions may have one more excluded values, or values that the variable cannot be or the expression would be undefined.

Example 1.

$$\begin{array}{rl}
\text{State the excluded value}(s)\text{:}\ \dfrac{x^2-1}{3x^2+5x} & \text{Denominator can't be zero} \\
3x^2+5x \neq 0 & \text{Factor} \\
x(3x+5) \neq 0 & \text{Set each factor not equal to zero} \\
x \neq 0 \ \text{ or } \ 3x+5 \neq 0 & \text{Subtract 5 from second equation} \\
\underline{-5\ -5} & \\
3x \neq -5 & \text{Divide by 3} \\
\overline{3} \quad \overline{3} & \\
x \neq \dfrac{-5}{3} & \text{Second equation is solved} \\
x \neq 0 \ \text{ or } \ \dfrac{-5}{3} & \text{Our Solution}
\end{array}$$

This means we can use any value for x in the equation except for 0 and $\frac{-5}{3}$. We can however, evaluate any other value in the expression. Rational expressions are easily evaluated by simplify substituting the value for the variable and using order of operations.

Example 2.

$$\frac{x^2-4}{x^2+6x+8} \text{ when } x=-6 \quad \text{Substitute} -5 \text{ in for each variable}$$

$$\frac{(-6)^2-4}{(-6)^2+6(-6)+8} \quad \text{Exponents first}$$

$$\frac{36-4}{36+6(-6)+8} \quad \text{Multiply}$$

$$\frac{36-4}{36-36+6} \quad \text{Add and subtract}$$

$$\frac{32}{6} \quad \text{Reduce, dividing by 2}$$

$$\frac{16}{3} \quad \text{Our Solution}$$

Just as we reduced the previous example, often a rational expression can be reduced, even without knowing the value of the variable. When we reduce we divide out common factors. We have already seen this with monomials when we discussed properties of exponents. If the problem only has monomials we can reduce the coefficents, and subtract exponents on the variables.

Example 3.

$$\frac{15x^4y^2}{25x^2y^6} \quad \text{Reduce , subtract exponents. Negative exponents move to denominator}$$

$$\frac{3x^2}{5y^4} \quad \text{Our Solution}$$

However, if there is more than just one term in either the numerator or denominator, we can't divide out common factors unless we first factor the numerator and denominator.

Example 4.

$$\frac{28}{8x^2-16} \quad \text{Denominator has } a \text{ common factor of } 8$$

$$\frac{28}{8(x^2-2)} \quad \text{Reduce by dividing } 24 \text{ and } 8 \text{ by } 4$$

$$\frac{7}{2(x^2-2)} \quad \text{Our Solution}$$

Example 5.

$$\frac{9x-3}{18x-6} \quad \text{Numerator has } a \text{ common factor of } 3\text{, denominator of } 6$$

$$\frac{3(3x-1)}{6(3x-1)} \quad \text{Divide out common factor } (3x-1) \text{ and divide } 3 \text{ and } 6 \text{ by } 3$$

$$\frac{1}{2} \quad \text{Our Solution}$$

Example 6.

$$\frac{x^2-25}{x^2+8x+15} \quad \text{Numerator is difference of squares, denominator is factored using ac}$$

$$\frac{(x+5)(x-5)}{(x+3)(x+5)} \quad \text{Divide out common factor } (x+5)$$

$$\frac{x-5}{x+3} \quad \text{Our Solution}$$

It is important to remember we cannot reduce terms, only factors. This means if there are any $+$ or $-$ between the parts we want to reduce we cannot. In the previous example we had the solution $\frac{x-5}{x+3}$, we cannot divide out the x's because they are terms (separated by $+$ or $-$) not factors (separated by multiplication).

Practice - Reduce Rational Expressions

State the excluded values for each.

1) $\frac{3k^2+30k}{k+10}$

2) $\frac{27p}{18p^2-36p}$

3) $\frac{15n^2}{10n+25}$

4) $\frac{x+10}{8x^2+80x}$

5) $\frac{10m^2+8m}{10m}$

6) $\frac{10x+16}{6x+20}$

7) $\frac{r^2+3r+2}{5r+10}$

8) $\frac{6n^2-21n}{6n^2+3n}$

9) $\frac{b^2+12b+32}{b^2+4b-32}$

10) $\frac{10v^2+30v}{35v^2-5v}$

Simplify each expression.

11) $\frac{21x^2}{18x}$

12) $\frac{12n}{4n^2}$

13) $\frac{24a}{40a^2}$

14) $\frac{21k}{24k^2}$

15) $\frac{32x^3}{8x^4}$

16) $\frac{90x^2}{20x}$

17) $\frac{18m-24}{60}$

18) $\frac{10}{81n^3+36n^2}$

19) $\frac{20}{4p+2}$

20) $\frac{n-9}{9n-81}$

21) $\frac{x+1}{x^2+8x+7}$

22) $\frac{28m+12}{36}$

23) $\frac{32x^2}{28x^2+28x}$

24) $\frac{49r+56}{56r}$

25) $\frac{n^2+4n-12}{n^2-7n+10}$

26) $\frac{b^2+14b+48}{b^2+15b+56}$

27) $\frac{9v+54}{v^2-4v-60}$

28) $\frac{30x-90}{50x+40}$

29) $\frac{12x^2-42x}{30x^2-42x}$

30) $\frac{k^2-12k+32}{k^2-64}$

31) $\frac{6a-10}{10a+4}$

32) $\frac{9p+18}{p^2+4p+4}$

33) $\frac{2n^2+19n-10}{9n+90}$

34) $\frac{3x^2-29x+40}{5x^2-30x-80}$

35) $\frac{9m+16}{20m-12}$

36) $\frac{9r^2+81r}{5r^2+50r+45}$

37) $\frac{2x^2-10x+8}{3x^2-7x+4}$

38) $\frac{50b-80}{50b+20}$

39) $\frac{7n^2-32n+16}{4n-16}$

40) $\frac{35v+35}{21v+7}$

41) $\frac{n^2-2n+1}{6n+6}$

42) $\frac{56x-48}{24x^2+56x+32}$

43) $\frac{7a^2-26a-45}{6a^2-34a+20}$

44) $\frac{4k^3-2k^2-2k}{9k^3-18k^2+9k}$

7.2

Rational Expressions - Multiply & Divide

Multiplying and dividing rational expressions is very similar to the process we use to multiply and divide fractions.

Example 7.

$\frac{15}{49}\cdot\frac{14}{45}$ First reduce common factors from numerator and denominator (5 and 7)

$\frac{3}{7}\cdot\frac{2}{9}$ Multiply numerators across and denominators across

$\frac{6}{63}$ Our Solution

The process is identical for division with the extra first step of multiplying by the reciprocal. When multiplying with rational expressions we follow the same process, first divide out common factors, then multiply straight across.

Example 8.

$\frac{25x^2}{9y^8}\cdot\frac{24y^4}{55x^7}$ Reduce coefficients by dividing out common factors (3 and 5)
Reduce, subtracting exponents, negative exponents in denominator

$\frac{5}{3y^4}\cdot\frac{8}{11x^5}$ Multiply across

$\frac{40}{33x^5y^4}$ Our Solution

Division is identical in process with the extra first step of multiplying by the reciprocal.

Example 9.

$\frac{a^4b^2}{a}\div\frac{b^4}{4}$ Multiply by the reciprocal

$\frac{a^4b^2}{a}\cdot\frac{4}{b^4}$ Subtract exponents on variables, negative exponents in denominator

$$\frac{a^3}{1}\cdot\frac{4}{b^2}\quad \text{Multiply across}$$

$$\frac{4a^3}{b^2}\quad \text{Our Solution}$$

Just as with reducing rational expressions, before we reduce a multiplication problem, it must be factored first.

Example 10.

$$\frac{x^2-9}{x^2+x-20}\cdot\frac{x^2-8x+16}{3x+9}\quad \text{Factor each numerator and denominator}$$

$$\frac{(x+3)(x-3)}{(x-4)(x+5)}\cdot\frac{(x-4)(x-4)}{3(x+3)}\quad \text{Divide out common factors } (x+3) \text{ and } (x-4)$$

$$\frac{x-3}{x+5}\cdot\frac{x-4}{3}\quad \text{Multiply across}$$

$$\frac{(x-3)(x-4)}{3(x+5)}\quad \text{Our Solution}$$

Again we follow the same pattern with division with the extra first step of multiplying by the reciprocal.

Example 11.

$$\frac{x^2-x-12}{x^2-2x-8}\div\frac{5x^2+15x}{x^2+x-2}\quad \text{Multiply by the reciprocal}$$

$$\frac{x^2-x-12}{x^2-2x-8}\cdot\frac{x^2+x-2}{5x^2+15x}\quad \text{Factor each numerator and denominator}$$

$$\frac{(x-4)(x+3)}{(x+2)(x-4)}\cdot\frac{(x+2)(x-1)}{5x(x+3)}\quad \text{Divide out common factors:}$$

$(x-4)$ and $(x+3)$ and $(x+2)$

$$\frac{1}{1}\cdot\frac{x-1}{5x}\quad \text{Multiply across}$$

$$\frac{x-1}{5x}\quad \text{Our Solution}$$

Practice - Multiply / Divide Rational Expressions

Simplify each expression.

1) $\frac{8x^2}{9}\cdot\frac{9}{2}$

2) $\frac{8x}{3x}\div\frac{4}{7}$

3) $\frac{9n}{2n}\cdot\frac{7}{5n}$

4) $\frac{9m}{5m^2}\cdot\frac{7}{2}$

5) $\frac{5x^2}{4}\cdot\frac{6}{5}$

6) $\frac{10p}{5}\div\frac{8}{10}$

7) $\frac{7(m-6)}{m-6}\cdot\frac{5m(7m-5)}{7(7m-5)}$

8) $\frac{7}{10(n+3)}\div\frac{n-2}{(n+3)(n-2)}$

9) $\frac{7r}{7r(r+10)}\div\frac{r-6}{(r-6)^2}$

10) $\frac{6x(x+4)}{x-3}\cdot\frac{(x-3)(x-6)}{6x(x-6)}$

11) $\frac{25n+25}{5}\cdot\frac{4}{30n+30}$

12) $\frac{9}{b^2-b-12}\div\frac{b-5}{b^2-b-12}$

13) $\frac{x-10}{35x+21}\div\frac{7}{35x+21}$

14) $\frac{v-1}{4}\cdot\frac{4}{v^2-11v+10}$

15) $\frac{x^2-6x-7}{x+5}\cdot\frac{x+5}{x-7}$

16) $\frac{1}{a-6}\cdot\frac{8a+80}{8}$

17) $\frac{8k}{24k^2-40k}\div\frac{1}{15k-25}$

18) $\frac{p-8}{p^2-12p+32}\div\frac{1}{p-10}$

19) $(n-8)\cdot\frac{6}{10n-80}$

20) $\frac{x^2-7x+10}{x-2}\cdot\frac{x+10}{x^2-x-20}$

21) $\frac{4m+36}{n+9}\cdot\frac{m-5}{5m^2}$

22) $\frac{2r}{r+6}\div\frac{2r}{7r+42}$

23) $\frac{3x-6}{12x-24}(x+3)$

24) $\frac{2n^2-12n-54}{n+7}\div(2n+6)$

25) $\frac{b+2}{40b^2-24b}(5b-3)$

26) $\frac{21v^2+16v-16}{3v+4}\div\frac{35v-20}{v-9}$

27) $\frac{n-7}{6n-12}\cdot\frac{12-6n}{n^2-13n+42}$

28) $\frac{x^2+11x+24}{6x^3+18x^2}\cdot\frac{6x^3+6x^2}{x^2+5x-24}$

29) $\frac{27a+36}{9a+63}\div\frac{6a+8}{2}$

30) $\frac{k-7}{k^2-k-12}\cdot\frac{7k^2-28k}{8k^2-56k}$

31) $\frac{x^2-12x+32}{x^2-6x-16}\cdot\frac{7x^2+14x}{7x^2+21x}$

32) $\frac{9x^3+54x^2}{x^2+5x-14}\cdot\frac{x^2+5x-14}{10x^2}$

33) $(10m^2+100m)\cdot\frac{18m^3-36m^2}{20m^2-40m}$

34) $\frac{n-7}{n^2-2n-35}\div\frac{9n+54}{10n+50}$

35) $\frac{7p^2+25p+12}{6p+48}\cdot\frac{3p-8}{21p^2-44p-32}$

36) $\frac{7x^2-66x+80}{49x^2+7x-72}\div\frac{7x^2+39x-70}{49x^2+7x-72}$

37) $\frac{10b^2}{30b+20}\cdot\frac{30b+20}{2b^2+10b}$

38) $\frac{35n^2-12n-32}{49n^2-91n+40}\cdot\frac{7n^2+16n-15}{5n+4}$

39) $\frac{7r^2-53r-24}{7r+2}\div\frac{49r+21}{49r+14}$

40) $\frac{12x+24}{10x^2+34x+28}\cdot\frac{15x+21}{5}$

7.3

Rational Expressions - LCD

As with fractions, the least common denominator or LCD is very important to working with rational expressions. The process we use to find and LCD is based on the process used to find the LCD of intergers.

Example 12.

Find the LCD of 8 and 6	Consider multiples of the larger number
$8, 16, 24\ldots.$	24 is the first multiple of 8 that is also divisible by 6
24	Our Solution

When finding the LCD of several monomials we first find the LCD of the coefficients, then use all variables and attach the highest exponent on each variable.

Example 13.

Find the LCD of $4x^2y^5$ and $6x^4y^3z^6$

	First find the LCD of coefficients 4 and 6
12	12 is the LCD of 4 and 6
$x^4y^5z^6$	Use all variable with highest exponents oneach variable
$12x^4y^5z^6$	Our Solution

The same pattern can be used on polynomials that have more than one term. However, we must first factor each polynomial so we can identify all the factors to be used (attaching highest exponent if necessary).

Example 14.

Find the LCD of x^2+2x-3 and x^2-x+12	Factor each polynomial
$(x-1)(x+3)$ and $(x-4)(x+3)$	LCD uses all unique factors
$(x-1)(x+3)(x-4)$	Our Solution

Notice we only used $(x+3)$ once in our LCD. This is because it only appears as a factor once in either polynomial. The only time we need to repeate a factor or use an exponent on a factor is if there are exponents when one of the polynomials is factored

Example 15.

Find the LCD of $x^2-10x+25$ and $x^2-14x+45$

	Factor each polynomial
$(x-5)^2$ and $(x-5)(x-9)$	LCD uses all unique factors with highest exponent
$(x-5)^2(x-9)$	Our Solution

The previous example could have also been done with factoring the first polynomial to $(x-5)(x-5)$. Then we would have used $(x-5)$ twice in the LCD because it showed up twice in one of the polynomials. However, it is the author's suggestion to use the exponents in factored form so as to use the same pattern (highest exponent) as used with monomials.

Once we know the LCD, our goal will be to build up fractions so they have matching denominators. In this lesson we will not be adding and subtracting fractions, just building them up to a common denominator. We can build up a fraction's denominator by multipliplying the numerator and denoinator by any factors that are not already in the denominator.

Example 16.

$\dfrac{5a}{3a^2b}=\dfrac{?}{6a^5b^3}$	Idenfity what factors we need to match denominators
$2a^3b^2$	$3\cdot 2=6$ and we need three more $a's$ and two more $b's$
$\dfrac{5a}{3a^2b}\left(\dfrac{2a^3b^2}{2a^3b^2}\right)$	Multiply numerator and denominator by this
$\dfrac{10a^3b^2}{6a^5b^3}$	Our Solution

Example 17.

$\dfrac{x-2}{x+4}=\dfrac{?}{x^2+7x+12}$ $(x+4)(x+3)$	Factor to idenfity factors we need to match denominators
$(x+3)$	The missing factor
$\dfrac{x-2}{x+4}\left(\dfrac{x+3}{x+3}\right)$	Multiply numerator and denominator by missing factor, FOIL numerator
$\dfrac{x^2+x-6}{(x+4)(x+3)}$	Our Solution

As the above example illustrates, we will multiply out our numerators, but keep our denominators factored. The reason for this is to add and subtract fractions we will want to be able to combine like terms in the numerator, then when we reduce at the end we will want our denominators factored.

Once we know how to find and LCD and how to build up fractions to a desired denominator we can combine them together by finding a common denominator and building up those fractions.

Example 18.

Build up each fraction so they have a common denominator

$$\frac{5a}{4b^3c} \text{ and } \frac{3c}{6a^2b} \qquad \text{First idenfity LCD}$$

$$12a^2b^3c \qquad \text{Determine what factors each fraction is missing}$$

$$\text{First: } 3a^2 \quad \text{Second: } 2b^2c \qquad \text{Multiply each fraction by missing factors}$$

$$\frac{5a}{4b^3c}\left(\frac{3a^2}{3a^2}\right) \text{ and } \frac{3c}{6a^2b}\left(\frac{2b^2c}{2b^2c}\right)$$

$$\frac{15a^3}{12a^2b^3c} \text{ and } \frac{6b^2c^2}{12a^2b^3c} \qquad \text{Our Solution}$$

Example 19.

Build up each fraction so they have a common denominator

$$\frac{5x}{x^2-5x-6} \text{ and } \frac{x-2}{x^2+4x+3} \qquad \text{Factor to find LCD}$$

$$(x-6)(x+1) \qquad (x+1)(x+3) \qquad \text{Use factors to find LCD}$$

$$\text{LCD: } (x-6)(x+1)(x+3) \qquad \text{Identify which factors are missing}$$

$$\text{First: } (x+3) \quad \text{Second: } (x-6) \qquad \text{Multiply fractions by missing factors}$$

$$\frac{5x}{(x-6)(x+1)}\left(\frac{x+3}{x+3}\right) \text{ and } \frac{x-2}{(x+1)(x+3)}\left(\frac{x-6}{x-6}\right) \qquad \text{Multiply numerators}$$

$$\frac{5x^2+15x}{(x-6)(x+1)(x+3)} \text{ and } \frac{x^2-8x+12}{(x-6)(x+1)(x+3)} \qquad \text{Our Solution}$$

Practice - Least Common Denominator

Build up denominators.

1) $\frac{3}{8} = \frac{?}{48}$

2) $\frac{a}{5} = \frac{?}{5a}$

3) $\frac{a}{x} = \frac{?}{xy}$

4) $\frac{5}{2x^2} = \frac{?}{8x^3y}$

5) $\frac{2}{3a^3b^2c} = \frac{?}{9a^5b^2c^4}$

6) $\frac{4}{x+5} = \frac{?}{9a^5b^2c^4}$

7) $\frac{2}{x+4} = \frac{?}{x^2-16}$

8) $\frac{x+1}{x-3} = \frac{?}{x^2-6x+9}$

9) $\frac{x-4}{x+2} = \frac{?}{x^2+5x+6}$

10) $\frac{x-6}{x+3} = \frac{?}{x^2-2x-15}$

Find Least Common Denominators

11) $2a^3, 6a^4b^2, 4a^3b^5$

12) $5x^2y, 25x^3y^5z$

13) $x^2-3x, x-3, x$

14) $4x-8, , x-2, 4$

15) $x+2, x-4$

16) $x, x-7, x+1$

17) $x^2-25, x+5$

18) x^2-9, x^2-3x+9

19) x^2+3x+2, x^2+5x+6

20) $x^2-7x+10, x^2-2x-15, x^2+x-6$

Find LCD and build up each fraction

21) $\frac{3a}{5b^2}, \frac{2}{10a^3b}$

22) $\frac{3x}{x-4}, \frac{2}{x+2}$

23) $\frac{x+2}{x-3}, \frac{x-3}{x+2}$

24) $\frac{5}{x^2-6x}, \frac{2}{x}, \frac{-3}{x-6}$

25) $\frac{x}{x^2-16}, \frac{3x}{x^2-8x+16}$

26) $\frac{5x+1}{x^2-3x-10}, \frac{4}{x-5}$

27) $\frac{x+1}{x^2-36}, \frac{2x+3}{x^2+12x+36}$

28) $\frac{3x+1}{x^2-x-6}, \frac{2x}{x^2+4x+3}$

29) $\frac{4x}{x^2-x-6}, \frac{x+2}{x-3}$

30) $\frac{3x}{x^2-6x+8}, \frac{x-2}{x^2+x-20}, \frac{5}{x^2+3x-10}$

7.4

Rational Expressions - Add & Subtract

Adding and subtracting rational expressions is identical to adding and subtracting with integers. Recall that when adding with a common denominator we add the numerators and keep the denominator. This is the same process used with rational expression. Remember to reduce, if possible, your final answer.

Example 20.

$$\frac{x-4}{x^2-2x-8}+\frac{x+8}{x^2-2x-8} \quad \text{Same denominator, add numerators, combine like terms}$$

$$\frac{2x+4}{x^2-2x-8} \quad \text{Factor numerator and denominator}$$

$$\frac{2(x+2)}{(x+2)(x-4)} \quad \text{Divide out } (x+2)$$

$$\frac{2}{x-4} \quad \text{Our Solution}$$

Subtraction with common denominator follows the same pattern, though the subtraction can cause problems if we are not careful with it. To avoid sign errors we will first distribute the subtraction through the numerator. Then we can treat it like an addition problem. This process is the same as "add the opposite" we saw when subtracting with negatives.

Example 21.

$$\frac{6x-12}{3x-6}-\frac{15x-6}{3x-6} \quad \text{Add the opposite of the second fraction (distribute negative)}$$

$$\frac{6x-12}{3x-6}+\frac{-15x+6}{3x-6} \quad \text{Add numerators, combine like terms}$$

$$\frac{-9x-6}{3x-6} \quad \text{Factor numerator and denominator}$$

$$\frac{-3(3x+2)}{3(x-2)} \quad \text{Divide out common factor of 3}$$

$$\frac{-(3x+2)}{x-2} \quad \text{Our Solution}$$

When we don't have a common denominator we will have to find the least common denominator (LCD) and build up each fraction so the denominators match. The following example shows this process with integers.

Example 22.

$$\frac{5}{6}+\frac{1}{4} \quad \text{The LCD is 12. Build up, multiply 6 by 2 and 4 by 3}$$

$$\left(\frac{2}{2}\right)\frac{5}{6}+\frac{1}{4}\left(\frac{3}{3}\right) \quad \text{Multiply}$$

$$\frac{10}{12}+\frac{3}{12} \quad \text{Add numerators}$$

$$\frac{13}{12} \quad \text{Our Solution}$$

The same process is used with variables.

Example 23.

$$\frac{7a}{3a^2b}+\frac{4b}{6ab^4} \quad \text{The LCD is } 6a^2b^4\text{. We will then build up each fraction}$$

$$\left(\frac{2b^3}{2b^3}\right)\frac{7a}{3a^2b}+\frac{4b}{6ab^4}\left(\frac{a}{a}\right) \quad \text{Multiply first fraction by } 2b^3 \text{ and second by } a$$

$$\frac{14ab^3}{6a^2b^4}+\frac{4ab}{6a^2b^4} \quad \text{Add numerators, no like terms to combine}$$

$$\frac{14ab^3+4ab}{6a^2b^4} \quad \text{Factor numerator}$$

$$\frac{2ab(7b^3+2)}{6a^2b^4} \quad \text{Reduce, dividing out factors } 2, a, \text{ and } b$$

$$\frac{7b^3+2}{3ab^3} \quad \text{Our Solution}$$

The same process can be used for subtraction, we will simply add the first step of adding the opposite.

Example 24.

$$\frac{4}{5a}-\frac{7b}{4a^2} \quad \text{Add the opposite}$$

$$\frac{4}{5a}+\frac{-7b}{4a^2} \quad \text{LCD is } 20a^2\text{. Build up denominators}$$

$$\left(\frac{4a}{4a}\right)\frac{4}{5a}+\frac{-7b}{4a^2}\left(\frac{5}{5}\right) \quad \text{Multiply first fraction by } 4a\text{, second by } 5$$

$$\frac{16a-35b}{20a^2} \quad \text{Our Solution}$$

If our denominators have more than one term in them we will need to factor first to find the LCD. Then we build up each denominator using the factors that are missing on each fraction.

Example 25.

$$\frac{6}{8a+4}+\frac{3a}{8} \quad \text{Factor denominators to find LCD}$$

$$4(2a+1) \quad 8 \quad \text{LCD is } 8(2a+1)\text{, build up each fraction}$$

$$\left(\frac{2}{2}\right)\frac{6}{4(2a+1)}+\frac{3a}{8}\left(\frac{2a+1}{2a+1}\right) \quad \text{Multiply first fraction by 2, second by } 2a+1$$

$$\frac{12}{8(2a+1)}+\frac{6a^2+3a}{8(2a+1)} \quad \text{Add numerators}$$

$$\frac{6a^2+3a+12}{8(2a+1)} \quad \text{Our Solution}$$

With subtraction remember to add the opposite.

Example 26.

$$\frac{x+1}{x-4}-\frac{x+1}{x^2-7x+12} \quad \text{Add the opposite (distribute negative)}$$

$$\frac{x+1}{x-4}+\frac{-x-1}{x^2-7x+12} \quad \text{Factor denominators to find LCD}$$

$$x-4 \quad (x-4)(x-3) \quad \text{LCD is } (x-4)(x-3)\text{, build up each fraction}$$

$$\left(\frac{x-3}{x-3}\right)\frac{x+1}{x-4}+\frac{-x-1}{x^2-7x+12} \quad \text{Only first fraction needs to be multiplied by } x-3$$

$$\frac{x^2-2x-3}{(x-3)(x-4)}+\frac{-x-1}{(x-3)(x-4)} \quad \text{Add numerators, combine like terms}$$

$$\frac{x^2-3x-4}{(x-3)(x-4)} \quad \text{Factor numerator}$$

$$\frac{(x-4)(x+1)}{(x-3)(x-4)} \quad \text{Divide out } x-4 \text{ factor}$$

$$\frac{x+1}{x-3} \quad \text{Our Solution}$$

Practice - Adding Rational Expressions

Add or subtract the rational expressions. Simplify your answers whenever possible.

1) $\frac{2}{a+3}+\frac{4}{a+3}$

2) $\frac{x^2}{x-2}-\frac{6x-8}{x-2}$

3) $\frac{t^2+4t}{t-1}+\frac{2t-7}{t-1}$

4) $\frac{a^2+3a}{a^2+5a-6}-\frac{4}{a^2+5a-6}$

5) $\frac{2x^2+3}{x^2-6x+5}-\frac{x^2-5x+9}{x^2-6x+5}$

6) $\frac{3}{x}+\frac{4}{x^2}$

7) $\frac{5}{6r}-\frac{5}{8r}$

8) $\frac{7}{xy^2}+\frac{3}{x^2y}$

9) $\frac{8}{9y^3}=\frac{5}{6t^2}$

10) $\frac{x+5}{8}+\frac{x-3}{12}$

11) $\frac{a+2}{2}-\frac{a-4}{4}$

12) $\frac{2a-1}{3a^2}+\frac{5a+1}{9a}$

13) $\frac{x-1}{4x}-\frac{2x+3}{x}$

14) $\frac{2c-d}{c^2d}-\frac{c+d}{cd^2}$

15) $\frac{5x+3y}{2x^2y}-\frac{3x+4y}{xy^2}$

16) $\frac{2}{x-1}+\frac{2}{x+1}$

17) $\frac{2z}{z-1}-\frac{3z}{z+1}$

18) $\frac{2}{x-5}+\frac{3}{4x}$

19) $\frac{8}{x^2-4}-\frac{3}{x+2}$

20) $\frac{4x}{x^2-25}+\frac{x}{x+5}$

21) $\frac{t}{t-3}-\frac{5}{4t-12}$

22) $\frac{2}{x+3}+\frac{4}{(x+3)^2}$

23) $\frac{2}{5x^2+5x}-\frac{4}{3x+3}$

24) $\frac{3a}{4a-20}+\frac{91}{6a-30}$

25) $\frac{t}{y-t}-\frac{y}{y+t}$

26) $\frac{x}{x-5}+\frac{x-5}{x}$

27) $\frac{x}{x^2+5x+6}-\frac{2}{x^2+3x+2}$

28) $\frac{2x}{x^2-1}-\frac{3}{x^2+5x+4}$

29) $\frac{x}{x^2+15x+56}-\frac{7}{x^2+13x+42}$

30) $\frac{2x}{x^2-9}+\frac{5}{x^2+x-6}$

31) $\frac{5x}{x^2-x-6}-\frac{18}{x^2-9}$

32) $\frac{4x}{x^2-2x-3}-\frac{3}{x^2-5x+6}$

33) $\frac{2x}{x^2-1}-\frac{4}{x^2+2x+3}$

34) $\frac{x-1}{x^2+3x+2}+\frac{x+5}{x^2+4x+3}$

35) $\frac{x+1}{x^2+2x-35}+\frac{x+6}{x^2+7x+10}$

36) $\frac{3x+2}{3x+6}+\frac{x}{4-x^2}$

37) $\frac{4-a^2}{a^2-9}-\frac{a-2}{3-a}$

38) $\frac{4y}{y^2-1}-\frac{2}{y}-\frac{2}{y+1}$

39) $\frac{2x}{1-2z}+\frac{3z}{2z+1}-\frac{3}{4z^2-1}$

40) $\frac{2r}{r^2-s^2}+\frac{1}{r+s}-\frac{1}{r-s}$

41) $\frac{2x-3}{x^2+3x+2}+\frac{3x-1}{x^2+5x+6}$

42) $\frac{x+2}{x^2-4x+3}+\frac{4x+5}{x^2+4x-5}$

43) $\frac{2x+7}{x^2-2x-3}-\frac{3x-2}{x^2+6x+5}$

44) $\frac{3x-8}{x^2+6x+8}+\frac{2x-3}{x^2+3x+2}$

7.5

Rational Expressions - Complex Fractions

Complex fractions have fractions in either the numerator, or denominator, or usually both. These fractions can be simplifed in one of two ways. This will be illustrated first with integers, then we will consider how the process can be expanded to include expressions with variables.

The first method uses order of operations to simplify the numerator and denominator first, then divide the two resulting fractions by multiplying by the reciprocal.

Example 27.

$$\frac{\frac{2}{3}-\frac{1}{4}}{\frac{5}{6}+\frac{1}{2}} \quad \text{Get common denominator in top and bottom fractions}$$

$$\frac{\frac{8}{12}-\frac{3}{12}}{\frac{5}{6}+\frac{3}{6}} \quad \text{Add and subtract fractions, reducing solutions}$$

$$\frac{\frac{5}{12}}{\frac{4}{3}} \quad \text{To divide fractions we multiply by the reciprocal}$$

$$\left(\frac{5}{12}\right)\left(\frac{3}{4}\right) \quad \text{Reduce}$$

$$\left(\frac{5}{4}\right)\left(\frac{1}{4}\right) \quad \text{Multiply}$$

$$\frac{5}{16} \quad \text{Our Solution}$$

The process above works just fine to simplify, but between getting common denominators, taking reciprocals, and reducing, it can be a very involved process. Generally we prefer a different method, to multiply the numerator and denominator of the large fraction (in effect each term in the complex fraction) by the least common denominator (LCD). This will allow us to reduce and clear the small fractions. We will simplify the same problem using this second method.

Example 28.

$$\frac{\frac{2}{3}-\frac{1}{4}}{\frac{5}{6}+\frac{1}{2}} \quad \text{LCD is 12, multiply each term}$$

$$\frac{\frac{2(12)}{3}-\frac{1(12)}{4}}{\frac{5(12)}{6}+\frac{1(12)}{2}} \quad \text{Reduce each fraction}$$

$$\frac{2(4)-1(3)}{5(2)+1(6)} \quad \text{Multiply}$$

$$\frac{8-3}{10+6} \quad \text{Add and subtract}$$

$$\frac{5}{16} \quad \text{Our Solution}$$

Clearly the second method is a much cleaner and faster method to arrive at our solution. It is the method we will use when simplifying with variables as well. We will first find the LCD of the small fractions, and multiply each term by this LCD so we can clear the small fractions and simplify.

Example 29.

$$\frac{1-\frac{1}{x^2}}{1-\frac{1}{x}} \quad \text{Identify LCD (use highest exponent)}$$

$$\text{LCD} = x^2 \quad \text{Multiply each term by LCD}$$

$$\frac{1(x^2)-\frac{1(x^2)}{x^2}}{1(x^2)-\frac{1(x^2)}{x}} \quad \text{Reduce fractions (subtract exponents)}$$

$$\frac{1(x^2)-1}{1(x^2)-x} \quad \text{Multiply}$$

$$\frac{x^2-1}{x^2-x} \quad \text{Factor}$$

$$\frac{(x+1)(x-1)}{x(x-1)} \quad \text{Divide out } (x-1) \text{ factor}$$

$$\frac{x+1}{x} \quad \text{Our Solution}$$

The process is the same if the LCD is a binomial, we will need to distribute

$$\frac{\frac{3}{x+4}-2}{5+\frac{2}{x+4}} \quad \text{Multiply each term by LCD, } (x+4)$$

$$\frac{\frac{3(x+4)}{x+4}-2(x+4)}{5(x+4)+\frac{2(x+4)}{x+4}} \quad \text{Reduce fractions}$$

$$\frac{3-2(x+4)}{5(x+4)+2} \quad \text{Distribute}$$

$$\frac{3-2x-8}{5x+20+2} \quad \text{Combine like terms}$$

$$\frac{-2x-5}{5x+22} \quad \text{Our Solution}$$

The more fractions we have in our problem, the more we repeate the same process.

Example 30.

$$\frac{\frac{2}{\mathrm{ab}^2}-\frac{3}{ab^3}+\frac{1}{ab}}{\frac{4}{a^2b}+ab-\frac{1}{ab}} \quad \text{Idenfity LCD (highest exponents}$$

$$\mathrm{LCD}=a^2b^3 \quad \text{Multiply each term by LCD}$$

$$\frac{\frac{2(a^2b^3)}{\mathrm{ab}^2}-\frac{3(a^2b^3)}{ab^3}+\frac{1(a^2b^3)}{ab}}{\frac{4(a^2b^3)}{a^2b}+ab(a^2b^3)-\frac{1(a^2b^3)}{ab}} \quad \text{Reduce each fraction (subtract exponents)}$$

$$\frac{2ab-3a+ab^2}{4b^2+a^3b^4-ab^2} \quad \text{Our Solution}$$

Some problems may require us to FOIL as we simplify. To avoid sign errors, if there is a binomial in the numerator, we will first distribute the negative through the numerator.

Example 31.

$$\frac{\frac{x-3}{x+3}-\frac{x+3}{x-3}}{\frac{x-3}{x+3}+\frac{x+3}{x-3}} \quad \text{Distribute the subtraction to numerator}$$

$$\frac{\frac{x-3}{x+3}+\frac{-x-3}{x-3}}{\frac{x-3}{x+3}+\frac{x+3}{x-3}} \quad \text{Idenfity LCD}$$

$$\text{LCD} = (x+3)(x-3) \quad \text{Multiply each term by LCD}$$

$$\frac{\frac{(x-3)(x+3)(x-3)}{x+3} + \frac{(-x-3)(x+3)(x-3)}{x-3}}{\frac{(x-3)(x+3)(x-3)}{x+3} + \frac{(x+3)(x+3)(x-3)}{x-3}} \quad \text{Reduce fractions}$$

$$\frac{(x-3)(x-3) + (-x-3)(x+3)}{(x-3)(x-3) + (x+3)(x-3)} \quad \text{FOIL}$$

$$\frac{x^2 - 6x + 9 - x^2 - 6x - 9}{x^2 - 6x + 9 + x^2 - 9} \quad \text{Combine like terms}$$

$$\frac{-12x}{x^2 - 6x} \quad \text{Factor out } x \text{ in denominator}$$

$$\frac{-12x}{x(x-6)} \quad \text{Divide out common factor } x$$

$$\frac{-12}{x-6} \quad \text{Our Solution}$$

If there are negative exponents in an expression we will have to first convert these negative exponents into fractions. Remember, the exponent is only on the factor it is attached to, not the whole term.

Example 32.

$$\frac{m^{-2} + 2m^{-1}}{m + 4m^{-2}} \quad \text{Make each negative exponent into } a \text{ fraction}$$

$$\frac{\frac{1}{m^2} + \frac{2}{m}}{m + \frac{4}{m^2}} \quad \text{Multiply each term by LCD, } m^2$$

$$\frac{\frac{1(m^2)}{m^2} + \frac{2(m^2)}{m}}{m(m^2) + \frac{4(m^2)}{m^2}} \quad \text{Reduce the fractions}$$

$$\frac{1 + 2m}{m^4 + 4} \quad \text{Our Solution}$$

Once we convert each negative exponent into a fraction, the problem solves exactly like the other complex fraction problems.

Practice - Complex Fractions

Solve.

1) $\dfrac{1+\frac{1}{x}}{1+\frac{1}{x^2}}$

3) $\dfrac{a-2}{\frac{4}{a}-a}$

5) $\dfrac{\frac{1}{a^2}-\frac{1}{a}}{\frac{1}{a^2}+\frac{1}{a}}$

7) $\dfrac{2-\frac{4}{x+2}}{5-\frac{10}{x+2}}$

9) $\dfrac{\frac{3}{2a-3}+2}{\frac{-6}{2a-3}-4}$

11) $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

13) $\dfrac{\frac{3}{x}}{\frac{9}{x^2}}$

15) $\dfrac{\frac{a^2-b^2}{4a^2b}}{\frac{a+b}{16\mathrm{ab}^2}}$

17) $\dfrac{1-\frac{3}{x}-\frac{10}{x^2}}{1+\frac{11}{x}+\frac{18}{x^2}}$

19) $\dfrac{1-\frac{2x}{3x-4}}{x-\frac{32}{3x-4}}$

21) $\dfrac{x-1+\frac{2}{x-4}}{x+3+\frac{6}{x-4}}$

23) $\dfrac{x-4+\frac{9}{2x+3}}{x+3-\frac{5}{2x+3}}$

25) $\dfrac{\frac{2}{b}-\frac{5}{b+3}}{\frac{3}{b}+\frac{3}{b+3}}$

27) $\dfrac{\frac{2}{b^2}-\frac{5}{\mathrm{ab}}-\frac{3}{a^2}}{\frac{2}{b^2}+\frac{7}{\mathrm{ab}}+\frac{3}{a^2}}$

29) $\dfrac{\frac{y}{y+2}-\frac{y}{y-2}}{\frac{y}{y+2}+\frac{y}{y-2}}$

2) $\dfrac{\frac{1}{y^2}-1}{1+\frac{1}{y}}$

4) $\dfrac{\frac{25}{a}-a}{5+a}$

6) $\dfrac{\frac{1}{b}+\frac{1}{2}}{\frac{4}{b^2-1}}$

8) $\dfrac{4+\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

10) $\dfrac{\frac{-5}{b-5}-3}{\frac{10}{b-5}+6}$

12) $\dfrac{\frac{2a}{a-1}-\frac{3}{a}}{\frac{-6}{a-1}-4}$

14) $\dfrac{\frac{x}{3x-2}}{\frac{x}{9x^2-4}}$

16) $\dfrac{1-\frac{1}{x}-\frac{6}{x^2}}{1-\frac{4}{x}+\frac{3}{x^2}}$

24) $\dfrac{\frac{1}{a}-\frac{3}{a-2}}{\frac{2}{a}+\frac{5}{a-2}}$

18) $\dfrac{\frac{15}{x^2}-\frac{2}{x}-1}{\frac{4}{x^2}-\frac{5}{x}+4}$

26) $\dfrac{\frac{1}{y^2}-\frac{1}{xy}-\frac{2}{x^2}}{\frac{1}{y^2}-\frac{3}{xy}+\frac{2}{x^2}}$

20) $\dfrac{1-\frac{12}{3x+10}}{x-\frac{8}{3x+10}}$

28) $\dfrac{\frac{x-1}{x+1}-\frac{x+1}{x-1}}{\frac{x-1}{x+1}+\frac{x+1}{x-1}}$

22) $\dfrac{x-5-\frac{18}{x+2}}{x+7+\frac{6}{x+2}}$

30) $\dfrac{\frac{x+1}{x-1}-\frac{1-x}{1+x}}{\frac{1}{(x+1)^2}+\frac{1}{(x-1)^2}}$

Simplify each of the following fractional expressions.

31) $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}}$

32) $\dfrac{x^{-2}y+xy^{-2}}{x^{-2}-y^{-2}}$

33) $\dfrac{x^{-3}y-xy^{-3}}{x^{-2}-y^{-2}}$

34) $\dfrac{4-4x^{-1}+x^{-2}}{4-x^{-2}}$

35) $\dfrac{x^{-2}-6x^{-1}+9}{x^2-9}$

36) $\dfrac{x^{-3}+y^{-3}}{x^{-2}-x^{-1}y^{-1}+y^{-2}}$

7.6

Rational Expressions - Proportions

When two fractions are equal, they are called a proportion. This definition can be generalized to two equal rational expressions. Proportions have an important property called the cross-product.

$$\textbf{Cross Product: If } \frac{\boldsymbol{a}}{\boldsymbol{b}} = \frac{\boldsymbol{c}}{\boldsymbol{d}} \textbf{ then ad} = \textbf{bc}$$

The cross product tells us we can multiply diagonally to get an equation with no fractions that we can solve.

Example 33.

$$\begin{array}{rl}
\frac{20}{6} = \frac{x}{9} & \text{Calculate cross product} \\
(20)(9) = 6x & \text{Multiply} \\
180 = 6x & \text{Divide both sides by 6} \\
\overline{6} \quad \overline{6} & \\
30 = x & \text{Our Solution}
\end{array}$$

If the proportion has more than one term in either numerator or denominator, we will have to distribute while calculatig the cross product.

Example 34.

$$\begin{array}{rl}
\frac{x+3}{4} = \frac{2}{5} & \text{Calculate cross product} \\
5(x+3) = (4)(2) & \text{Multiply and distribute} \\
5x + 15 = 8 & \text{Solve} \\
\underline{-15 - 15} & \text{Subtract 15 from both sides} \\
5x = -7 & \text{Divide both sides by 5} \\
\overline{5} \quad \overline{5} & \\
x = -\frac{7}{5} & \text{Our Solution}
\end{array}$$

This same idea can be seen when the variable appears in several parts of the proportion.

Example 35.

$$\begin{array}{rl}
\frac{4}{x} = \frac{6}{3x+2} & \text{Calculate cross product} \\
4(3x+2) = 6x & \text{Distribute} \\
12x + 8 = 6x & \text{Move variables to one side}
\end{array}$$

$$
\begin{array}{rl}
\underline{-12x \quad -12x} & \text{Subtract } 12x \text{ from both sides} \\
8 = -6x & \text{Divide both sides by } -6 \\
\overline{-6} \quad \overline{-6} & \\
-\frac{4}{3} = x & \text{Our Solution}
\end{array}
$$

Example 36.

$$
\begin{array}{rl}
\frac{2x-3}{7x+4} = \frac{2}{5} & \text{Calculate cross product} \\
5(2x-3) = 2(7x+4) & \text{Distribute} \\
10x - 15 = 14x + 8 & \text{Move variables to one side} \\
\underline{-10x \qquad -10x} & \text{Subtract } 10x \text{ from both sides} \\
-15 = 4x + 8 & \text{Subtract 8 from both sides} \\
\underline{-8 \qquad -8} & \\
-23 = 4x & \text{Divide both sides by 4} \\
\overline{4} \quad \overline{4} & \\
-\frac{23}{4} = x & \text{Our Solution}
\end{array}
$$

As we solve proportions we may end up with a quadratic that we will have to solve. We can solve this quadratic in the same way we solved quadratics in the past, either factoring, completing the square or the quadratic formula. As with solving quadratics before, we will generally end up with two solutions.

Example 37.

$$
\begin{array}{rl}
\frac{k+3}{3} = \frac{8}{k-2} & \text{Calculate cross product} \\
(k+3)(k-2) = (8)(3) & \text{FOIL and multiply} \\
k^2 + k - 6 = 24 & \text{Make equation equal zero} \\
\underline{-24 - 24} & \text{Subtract 24 from both sides} \\
k^2 + k - 30 = 0 & \text{Factor} \\
(k+6)(k-5) = 0 & \text{Set each factor equal to zero} \\
k + 6 = 0 \ \text{ or } \ k - 5 = 0 & \text{Solve each equation} \\
\underline{-6 - 6} \qquad \underline{+5 = 5} & \text{Add or subtract} \\
k = -6 \ \text{ or } \ k = 5 & \text{Our Solutions}
\end{array}
$$

Proportions are very useful in how they can be used in many different types of applications. We can use them to compare different quantities and make conclusions about how quantities are related. As we set up these problems it is important to remember to stay organized, if we are comparing dogs and cats, and the number of dogs is in the numerator of the first fraction, then the numerator of the second fraction should also refer to the dogs. This consistancy of the numerator and denominator is essential in setting up our proportions.

Example 38.

A six foot tall man casts a shadow that is 3.5 feet long. If the shadow of a flag pole is 8 feet long, how tall is the flag pole?

$$\frac{\text{shadow}}{\text{height}} \quad \text{We will put shadows in numerator, heights in denomintor}$$

$$\frac{3.5}{6} \quad \text{The man has } a \text{ shadow of 3.5 feet and } a \text{ height of 6 feet}$$

$$\frac{8}{x} \quad \text{The flagpole has } a \text{ shadow of 8 feet, but we don}'t \text{ know the height}$$

$$\begin{array}{rl}
\frac{3.5}{6} = \frac{8}{x} & \text{This gives us our proportion, calculate cross product} \\
3.5x = (8)(6) & \text{Multiply} \\
3.5x = 48 & \text{Divide both sides by 3.5} \\
\overline{3.5} \quad \overline{3.5} & \\
x = 13.7\,\text{ft} & \text{Our Solution}
\end{array}$$

Example 39.

In a basketball game, the home team was down by 9 points at the end of the game. They only scored 6 points for every 7 points the visiting team scored. What was the final score of the game?

$$\frac{\text{home}}{\text{visiter}} \quad \text{We willput home in numerator, visiter in denominator}$$

$$\frac{x-9}{x} \quad \text{Don}'t \text{ know visiter score, but home is 9 points less}$$

$$\frac{6}{7} \quad \text{Home team scored 6 for every 7 the visiter scored}$$

$$\begin{array}{rl}
\frac{x-9}{x} = \frac{6}{7} & \text{This gives our proportion, calculate the cross product} \\
7(x-9) = 6x & \text{Distribute} \\
7x - 63 = 6x & \text{Move variables to one side} \\
\underline{-7x \qquad -7x} & \text{Subtract } 7x \text{ from both sides} \\
-63 = -x & \text{Divide both sides by } -1 \\
\overline{-1} \quad \overline{-1} & \\
63 = x & \text{We used } x \text{ for the visiter score.} \\
63 - 9 = 54 & \text{Subtract 9 to get the home score} \\
63 \text{ to } 54 & \text{Our Solution}
\end{array}$$

Practice - Proportions

Solve each proportion.

1) $\frac{10}{a} = \frac{6}{8}$

2) $\frac{7}{9} = \frac{n}{6}$

3) $\frac{7}{6} = \frac{2}{k}$

4) $\frac{8}{x} = \frac{4}{8}$

5) $\frac{6}{x} = \frac{8}{2}$

6) $\frac{n-10}{8} = \frac{9}{3}$

7) $\frac{m-1}{5} = \frac{8}{2}$

8) $\frac{8}{5} = \frac{3}{x-8}$

9) $\frac{2}{9} = \frac{10}{p-4}$

10) $\frac{9}{n+2} = \frac{3}{9}$

11) $\frac{b-10}{7} = \frac{b}{4}$

12) $\frac{9}{4} = \frac{r}{r-4}$

13) $\frac{x}{5} = \frac{x+2}{9}$

14) $\frac{n}{8} = \frac{n-4}{3}$

15) $\frac{3}{10} = \frac{a}{a+2}$

16) $\frac{x+1}{9} = \frac{x+2}{2}$

17) $\frac{v-5}{v+6} = \frac{4}{9}$

18) $\frac{n+8}{10} = \frac{n-9}{4}$

19) $\frac{7}{x-1} = \frac{4}{x-6}$

20) $\frac{k+5}{k-6} = \frac{8}{5}$

21) $\frac{x+5}{5} = \frac{6}{x-2}$

22) $\frac{4}{x-3} = \frac{x+5}{5}$

23) $\frac{m+3}{4} = \frac{11}{m-4}$

24) $\frac{x-5}{8} = \frac{4}{x-1}$

25) $\frac{2}{p+4} = \frac{p+5}{3}$

26) $\frac{5}{n+1} = \frac{n-4}{10}$

27) $\frac{n+4}{3} = \frac{-3}{n-2}$

28) $\frac{1}{n+3} = \frac{n+2}{2}$

29) $\frac{3}{x+4} = \frac{x+2}{5}$

30) $\frac{x-5}{4} = \frac{3}{x+3}$

Answer each question. Round your answer to the nearest tenth. Round dollar amounts to the nearest cent.

31) The currency in Western Samoa is the Tala. The exchange rate is approximately \$0.70 to 1 Tala. At this rate, how many dollars would you get if you exchanged 13.3 Tala?

32) If you can buy one plantain for \$0.49 then how many can you buy with \$7.84?

33) Kali reduced the size of a painting to a height of 1.3 in. What is the new width if it was originally 5.2 in. tall and 10 in. wide?

34) A model train has a scale of 1.2 in : 2.9 ft. If the model train is 5 in tall then how tall is the real train?

35) A bird bath that is 5.3 ft tall casts a shadow that is 25.4 ft long. Find the length of the shadow that a 8.2 ft adult elephant casts.

36) Victoria and Georgetown are 36.2 mi from each other. How far apart would the cities be on a map that has a scale of 0.9 in : 10.5 mi?

37) The Vikings led the Timberwolves by 19 points at the half. If the Vikings scored 3 points for every 2 points the Timberwolves scored, what was the half time score?

38) Sarah worked 10 mroe hours than Josh. If Sarah worked 7 hr for every 2 hr Josh worked, how long did they each work?

39) Computer Services Inc. charges $8 more for a repair than Low Cost Computer Repair. If the ratio of the costs is 3 : 6, what will it cost for the repair at Low Cost Computer Repair?

40) Kelsey's commute is 15 minutes longer than Christina's. If Christina drives 12 minutes for every 17 minutes Kelsey drives, how long is each commute?

7.7

Rational Expressions - Rational Equations

When solving equations that are made up of rational expressions we will solve them using the same strategy we used to solve linear equations with fractions. When we solved problems like the next example, we cleared the fraction by multiplying by the least common denominator (LCD)

Example 40.

$$\begin{aligned}
\frac{2}{3}x-\frac{5}{6}&=\frac{3}{4} && \text{Multiply each term by LCD, 12}\\
\frac{2(12)}{3}x-\frac{5(12)}{6}&=\frac{3(12)}{4} && \text{Reduce fractions}\\
2(4)x-5(2)&=3(3) && \text{Multiply}\\
8x-10&=9 && \text{Solve}\\
\underline{+10}\;&\underline{+10} && \text{Add 10 to both sides}\\
8x&=19 && \text{Divide both sides by 8}\\
\overline{8}\;\;&\;\;\overline{8} && \\
x&=\frac{19}{8} && \text{Our Solution}
\end{aligned}$$

We will use the same process to solve rational equations, the only difference is our

LCD will be more involved. We will also have to beware of domain issues. If our LCD equals zero, the solution is undefined. We will always check our solutions in the LCD as we may have to remove a solution from our solution set.

Example 41.

$$\frac{5x+5}{x+2}+3x=\frac{x^2}{x+2} \quad \text{Multiply each term by LCD, } (x+2)$$

$$\frac{(5x+5)(x+2)}{x+2}+3x(x+2)=\frac{x^2(x+2)}{x+2} \quad \text{Reduce fractions}$$

$$\begin{array}{rl}
5x+5+3x(x+2)=x^2 & \text{Distribute}\\
5x+5+3x^2+6x=x^2 & \text{Combine like terms}\\
3x^2+11x+5=x^2 & \text{Make equation equal zero}\\
\underline{-x^2 \qquad\qquad -x^2} & \text{Subtract } x^2 \text{ from both sides}\\
2x^2+11x+5=0 & \text{Factor}\\
(2x+1)(x+5)=0 & \text{Set each factor equal to zero}\\
2x+1=0 \text{ or } x+5=0 & \text{Solve each equation}\\
\underline{-1-1} \qquad \underline{-5-5} & \\
2x=-1 \text{ or } x=-5 & \\
\overline{2} \quad \overline{2} \qquad\qquad & \\
x=-\frac{1}{2} \text{ or } -5 & \text{Check solutions, LCD can}'t \text{ be zero}\\
-\frac{1}{2}+2=\frac{3}{2} \qquad -5+2=-3 & \text{Neither make LCD zero, both are solutions}\\
x=-\frac{1}{2} \text{ or } -5 & \text{Our Solution}
\end{array}$$

The LCD can be several factors in these problems. As the LCD gets more complex, it is important to remember the process we are using to solve is still the same.

Example 42.

$$\frac{x}{x+2}+\frac{1}{x+1}=\frac{5}{(x+1)(x+2)} \quad \text{Multiply terms by LCD, } (x+1)(x+2)$$

$$\frac{x(x+1)(x+2)}{x+2}+\frac{1(x+1)(x+2)}{x+1}=\frac{5(x+1)(x+2)}{(x+1)(x+2)} \quad \text{Reduce fractions}$$

$$
\begin{array}{rl}
x(x+1)+1(x+2)=5 & \text{Distribute} \\
x^2+x+x+2=5 & \text{Combine like terms} \\
x^2+2x+2=5 & \text{Make equatino equal zero} \\
\underline{-5-5} & \text{Subtract 6 from both sides} \\
x^2+2x-3=0 & \text{Factor} \\
(x+3)(x-1)=0 & \text{Set each factor equal to zero} \\
x+3=0 \ \text{ or } \ x-1=0 & \text{Solve each equation} \\
\underline{-3-3} \qquad \underline{+1+1} & \\
x=-3 \ \text{ or } \ x=1 & \text{Check solutions, LCD can}'t \text{ be zero} \\
(-3+1)(-3+2)=(-2)(-1)=2 & \text{Check } -3 \text{ in } (x+1)(x+2), \text{it works} \\
(1+1)(1+2)=(2)(3)=6 & \text{Check 1 in } (x+1)(x+2), \text{it works} \\
x=-3 \ \text{ or } \ 1 & \text{Our Solution}
\end{array}
$$

In the previous example the denominators were factored for us. More often we will need to factor before finding the LCD

Example 43.

$$
\begin{array}{rl}
\dfrac{x}{x-1}-\dfrac{1}{x-2}=\dfrac{11}{x^2-3x+2} & \text{Factor denominator} \\
(x-1)(x-2) & \\
\text{LCD}=(x-1)(x-2) & \text{Identify LCD} \\
\dfrac{x(x-1)(x-2)}{x-1}-\dfrac{1(x-1)(x-2)}{x-2}=\dfrac{11(x-1)(x-2)}{(x-1)(x-2)} & \text{Multiply each term by LCD, reduce} \\
x(x-2)-1(x-1)=11 & \text{Distribute} \\
x^2-2x-x+1=11 & \text{Combine like terms} \\
x^2-3x+1=11 & \text{Make equation equal zero} \\
\underline{-11-11} & \text{Subtract 11 from both sides} \\
x^2-3x-10=0 & \text{Factor} \\
(x-5)(x+2)=0 & \text{Set each factor equal to zero} \\
x-5=0 \ \text{ or } \ x+2=0 & \text{Solve each equation} \\
\underline{+5+5} \qquad \underline{-2-2} & \\
x=5 \ \text{ or } \ x=-2 & \text{Check answers, LCD can}'t \text{ be 0} \\
(5-1)(5-2)=(4)(3)=12 & \text{Check 5 in } (x-1)(x-2), \text{it works} \\
(-2-1)(-2-2)=(-3)(-4)=12 & \text{Check } -2 \text{ in } (x-1)(x-2), \text{it works}
\end{array}
$$

$$x = 5 \text{ or } -2 \quad \text{Our Solution}$$

If we are subtracting a fraction in th problem, in may be easier to avoid a future sign error by first distributing the negative through the numerator.

Example 44.

$$\frac{x-2}{x-3} - \frac{x+2}{x+2} = \frac{5}{8} \quad \text{Distirbute negative through numerator}$$

$$\frac{x-2}{x-3} + \frac{-x-2}{x+2} = \frac{5}{8} \quad \text{Identify LCD, } 8(x-3)(x+2)\text{, multiply each term}$$

$$\frac{(x-2)8(x-3)(x+2)}{x-3} + \frac{(-x-2)8(x-3)(x+2)}{x+2} = \frac{5 \cdot 8(x-3)(x+2)}{8} \quad \text{Reduce}$$

$$\begin{aligned}
8(x-2)(x+2) + 8(-x-2)(x-3) &= 5(x-3)(x+2) && \text{FOIL} \\
8(x^2-4) + 8(-x^2+x+6) &= 5(x^2-x-6) && \text{Distribute} \\
8x^2 - 32 - 8x^2 + 8x + 48 &= 5x^2 - 5x - 30 && \text{Combine like terms} \\
8x + 16 &= 5x^2 - 5x - 30 && \text{Make equation equal zero} \\
\underline{-8x - 16} \qquad & \quad \underline{-8x - 16} && \text{Subtract } 8x \text{ and } 16 \\
0 &= 5x^2 - 13x - 46 && \text{Factor} \\
0 &= (5x-23)(x+2) && \text{Set each factor equal to zero} \\
5x - 23 = 0 &\text{ or } x + 2 = 0 && \text{Solve each equation} \\
\underline{+23 + 23} & \quad \underline{-2 - 2} && \\
5x = 23 &\text{ or } x = -2 && \\
\underline{5} \quad \underline{5} & && \\
x = \frac{23}{5} &\text{ or } -2 && \text{Check solutions, LCD can't be 0} \\
8\left(\frac{23}{5} - 3\right)\left(\frac{23}{5} + 2\right) = 8\left(\frac{8}{5}\right)\left(\frac{33}{5}\right) &= \frac{2112}{25} && \text{Check } \frac{23}{5} \text{ in } 8(x-3)(x+2)\text{, it works} \\
8(-2-3)(-2+2) = 8(-5)(0) &= 0 && \text{Check } -2 \text{ in } 8(x-3)(x+2)\text{, can't be 0!} \\
x &= \frac{23}{5} && \text{Our Solution}
\end{aligned}$$

In the previous example, one of the solutions we found made the LCD zero. When this happens we ignore this result and only use the results that make the rational expressions defined.

Practice - Rational Equations

Solve the following equations for the given variable:

1) $3x - \frac{1}{2} - \frac{1}{x} = 0$

3) $x + \frac{20}{x-4} = \frac{5x}{x-4} - 2$

5) $x + \frac{6}{x-3} = \frac{2x}{x-3}$

7) $\frac{2x}{3x-4} = \frac{4x+5}{6x-1} - \frac{3}{3x-4}$

9) $\frac{3m}{2m-5} - \frac{7}{3m+1} = \frac{3}{2}$

11) $\frac{4-x}{1-x} = \frac{12}{3-x}$

13) $\frac{7}{y-3} - \frac{1}{2} = \frac{y-2}{y-4}$

15) $\frac{1}{x+2} - \frac{1}{2-x} = \frac{3x+8}{x^2-4}$

17) $\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{5}{6}$

19) $\frac{3}{2x+1} + \frac{2x+1}{1-2x} = 1 - \frac{8x^2}{4x^2-1}$

21) $\frac{x-2}{x+3} - \frac{1}{x-2} = \frac{1}{x^2+x-6}$

23) $\frac{3}{x+2} + \frac{x-1}{x+5} = \frac{5x+20}{6x+24}$

25) $\frac{x}{x-1} - \frac{2}{x+1} = \frac{4x^2}{x^2-1}$

27) $\frac{2x}{x+1} - \frac{3}{x+5} = \frac{-8x^2}{x^2+6x+5}$

29) $\frac{x-5}{x-9} + \frac{x+3}{x-3} = \frac{-4x^2}{x^2 12x+27}$

31) $\frac{x+1}{x-4} + \frac{3x-2}{x+4} = \frac{7x^2}{x^2-16}$

33) $\frac{x+3}{x-2} + \frac{x-2}{x+1} = \frac{9x^2}{x^2-x-2}$

35) $\frac{3x-1}{x+6} - \frac{2x-3}{x-3} = \frac{-3x^2}{x^2+3x-18}$

39)

2) $x + 1 = \frac{4}{x+1}$

4) $\frac{x^2+6}{x-1} + \frac{x-2}{x-1} = 2x$

6) $\frac{4-x}{x-1} = \frac{12}{3-x} + 1$

8) $\frac{6x+5}{2x^2-2x} - \frac{2}{1-x^2} = \frac{3x}{x^2-1}$

10) $\frac{4x}{2x-6} - \frac{4}{5x-15} = \frac{1}{2}$

12) $\frac{7}{3-x} + \frac{1}{2} = \frac{3}{4-x}$

14) $\frac{2}{3-x} - \frac{6}{8-x} = 1$

16) $\frac{x+2}{3x-1} - \frac{1}{x} = \frac{3x+8}{3x^2-x}$

18) $\frac{x-1}{x-3} + \frac{x+2}{x+3} = \frac{3}{4}$

20) $\frac{3x-5}{5x-5} + \frac{5x-1}{7x-7} - \frac{x-4}{1-x} = 2$

22) $\frac{x-1}{x-2} + \frac{x+4}{2x+1} = \frac{1}{2x^2-3x-2}$

24) $\frac{x}{x+3} - \frac{4}{x-2} = \frac{-5x^2}{x^2+x-6}$

26) $\frac{2x}{x+2} + \frac{2}{x-4} = \frac{5x^2}{x^2-2x-8}$

28) $\frac{x}{x+1} - \frac{3}{x+3} = \frac{-2x^2}{x^2+4x+3}$

30) $\frac{x-3}{x+6} + \frac{x-2}{x-3} = \frac{x^2}{x^2+3x-18}$

32) $\frac{x-3}{x-6} + \frac{x+5}{x+3} = \frac{-2x^2}{x^2-3x-18}$

34) $\frac{4x+1}{x+3} + \frac{5x-3}{x-1} = \frac{8x^2}{x^2+2x-3}$

7.8

Rational Expressions - Dimensional Analysis

One application of rational expressions deals with converting units. When we convert units of measure we can do so by multiplying several fractions together in a process known as dimensional analysis. The trick will be to decide what fractions to multiply. When multiplying, if we multiply by 1, the value of the expression does not change. One written as a fraction can look like many different things as long as the numerator and denominator are identical in value. Notice the numerator and denominator are not identical in appearance, but rather identical in value. Below are several fractions, each equal to one where numerator and denominator are identical in value.

$$\frac{1}{1}=\frac{4}{4}=\frac{\frac{1}{2}}{\frac{2}{4}}=\frac{100\,\text{cm}}{1m}=\frac{1\,\text{lb}}{16\,\text{oz}}=\frac{1\,\text{hr}}{60\,\text{min}}=\frac{60\,\text{min}}{1\,\text{hr}}$$

The last few fractions that include units are called conversion factors. We can make a conversion factor out of any two measurements that represent the same distance. For example, 1 mile = 5280 feet. We could then make a conversion factor $\frac{1\,\text{mi}}{5280\,\text{ft}}$ because both values are the same, the fraction is still equal to one. Similarly we could make a conversion factor $\frac{5280\,\text{ft}}{1\,\text{mi}}$. The trick for conversions will be to use the correct fractions.

The idea behind dimensional analysis is we we multiply by a fraction in such a way that the units we don't want will divide out of the problem. We found out when multiplying rational expressions that if a variable appears in the numerator and denominator we can divide it out of the expression. It is the same with units. Consider the following conversion.

Example 45.

17.37 miles to feet	Write 17.37 miles as *a* fraction, put it over 1
$\left(\frac{17.37\,\text{mi}}{1}\right)$	To divide out the miles we need miles in the denominator
$\left(\frac{17.37\,\text{mi}}{1}\right)\left(\frac{??\,\text{ft}}{??\,\text{mi}}\right)$	We are converting to feet, so this will go in the numerator
$\left(\frac{17.37\,\text{mi}}{1}\right)\left(\frac{5280\,\text{ft}}{1\,\text{mi}}\right)$	Fill in the relationship described above, 1 mile = 5280 feet
$\left(\frac{17.37}{1}\right)\left(\frac{5280\,\text{ft}}{1}\right)$	Divide out the miles and multiply across
$91,713.6\,\text{ft}$	Our Solution

In the previous example, we had to use the conversion factor $\frac{5280\,\text{ft}}{1\,\text{mi}}$ so the miles would divide out. If we had used $\frac{1\,\text{mi}}{5280\,\text{ft}}$ we would not have been able to divide out the miles. This is why when doing dimensional analysis it is very important to use units in the set-up of the problem, so we know how to correctly set up the conversion factor.

Example 46.

If 1 pound = 16 ounces, how many pounds 435 ounces?

$$\left(\frac{435\,\text{oz}}{1}\right) \quad \text{Write 435 as } a \text{ fraction, put it over 1}$$

$$\left(\frac{435\,\text{oz}}{1}\right)\left(\frac{??\,\text{lbs}}{??\,\text{oz}}\right) \quad \text{To divide out oz,}$$

$$\text{put it in the denominator and lbs in numerator}$$

$$\left(\frac{435\,\text{oz}}{1}\right)\left(\frac{1\,\text{lbs}}{16\,\text{oz}}\right) \quad \text{Fill in the given relationship, 1 pound} = \text{16 ounces}$$

$$\left(\frac{435}{1}\right)\left(\frac{1\,\text{lbs}}{16}\right) = \frac{435\,\text{lbs}}{16} \quad \text{Divide out oz, multiply across. Divide result}$$

$$27.1875\,\text{lbs} \quad \text{Our Solution}$$

The same process can be used to convert problems with several units in them. Consider the following example.

Example 47.

A student averaged 45 miles per hour on a trip. What was the student's speed in feet per second?

$$\left(\frac{45\,\text{mi}}{\text{hr}}\right) \quad \text{"per" is the fraction bar, put hr in denominator}$$

$$\left(\frac{45\,\text{mi}}{\text{hr}}\right)\left(\frac{5280\,\text{ft}}{1\,\text{mi}}\right) \quad \text{To clear mi they must go in denominator and become ft}$$

$$\left(\frac{45\,\text{mi}}{\text{hr}}\right)\left(\frac{5280\,\text{ft}}{1\,\text{mi}}\right)\left(\frac{1\,\text{hr}}{3600\,\text{sec}}\right) \quad \text{To clear hr they must go in numerator and become sec}$$

$$\left(\frac{45}{1}\right)\left(\frac{5280\,\text{ft}}{1}\right)\left(\frac{1}{3600\,\text{sec}}\right) \quad \text{Divide out mi and hr. Multiply across}$$

$$\frac{237600\,\text{ft}}{3600\,\text{sec}} \quad \text{Divide numbers}$$

$$66\,\text{ft per sec} \quad \text{Our Solution}$$

If the units are two-dimensional (such as square inches - in^2) or thre-dimensional (such as cubic feet - ft^3) we will need to put the same exponent on the conversion factor. So if we are converting square inches (in^2) to square ft (ft^2), the conversion factor would be squared, $\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)^2$. Similarly if the units are cubed, we will cube the convesion factor.

Example 48.

Convert 8 cubic feet to yd^3	Write 8ft^3 as fraction, put it over 1
$\left(\frac{8\,\text{ft}^3}{1}\right)$	To clear ft, put them in denominator, yard in numerator
$\left(\frac{8\,\text{ft}^3}{1}\right)\left(\frac{??\text{yd}}{??\text{ft}}\right)^3$	Because the units are cubed, we cube the conversion factor
$\left(\frac{8\,\text{ft}^3}{1}\right)\left(\frac{1\,\text{yd}}{3\,\text{ft}}\right)^3$	Evaluate exponent, cubing all numbers and units
$\left(\frac{8\,\text{ft}^3}{1}\right)\left(\frac{1\,\text{yd}^3}{27\,\text{ft}^3}\right)$	Divide out ft^3
$\left(\frac{8}{1}\right)\left(\frac{1\,\text{yd}^3}{27}\right)=\frac{8\,\text{yd}^3}{27}$	Multiply across and divide
$0.296296\,\text{yd}^3$	Our Solution

When calculating area or volume, be sure to use the units and multiply them as well.

Example 49.

A room is 10 ft by 12 ft. How many square yards are in the room?

$A = lw = (10\text{ft})(12\,\text{ft}) = 120\,\text{ft}^2$	Multiply length by width, also multiply units
$\left(\frac{120\,\text{ft}^2}{1}\right)$	Write area as a fraction, put it over 1
$\left(\frac{120\,\text{ft}^2}{1}\right)\left(\frac{??\text{yd}}{??\text{ft}}\right)^2$	Put ft in denominator to clear, square conversion factor
$\left(\frac{120\,\text{ft}^2}{1}\right)\left(\frac{1\,\text{yd}}{3\,\text{ft}}\right)^2$	Evaluate exponent, squaring all numbers and units

$$\left(\frac{120\,\text{ft}^2}{1}\right)\left(\frac{1\,\text{yd}^2}{9\,\text{ft}^2}\right) \quad \text{Divide out ft}^2$$

$$\left(\frac{120}{1}\right)\left(\frac{1\,\text{yd}^2}{9}\right) = \frac{120\,\text{yd}^2}{9} \quad \text{Multiply across and divide}$$

$$13.33\,\text{yd}^2 \quad \text{Our solution}$$

To focus on the process of conversions, a conversion sheet has been included at the end of this lesson which includes several conversion factors for length, volume, mass and time in both English and Metric units.

The process of dimensional analysis can be used to convert other types of units as well. If we can identify relationships that represent the same value we can make them into a conversion factor.

Example 50.

A child is perscribed a dosage of 12 mg of a certain drug and is allowed to refill his perscription twice. If a there are 60 tablets in a perscription, and each tablet has 4 mg, how many doses are in the 3 perscriptions (original + 2 refills)?

$$\text{Convert 3 Rx to doses} \quad \text{Idenfity what problem is asking}$$

$$1\,\text{Rx} = 60\,\text{tab},\ 1\,\text{tab} = 4\,\text{mg},\ 1\,\text{dose} = 12\,\text{mg} \quad \text{Idenfity given conversion factors}$$

$$\left(\frac{3\,\text{Rx}}{1}\right) \quad \text{Write 3Rx as fraction, put over 1}$$

$$\left(\frac{3\,\text{Rx}}{1}\right)\left(\frac{60\,\text{tab}}{1\text{Rx}}\right) \quad \text{Convert Rx to tab, put Rx in denoimnator}$$

$$\left(\frac{3\,\text{Rx}}{1}\right)\left(\frac{60\,\text{tab}}{1\text{Rx}}\right)\left(\frac{4\,\text{mg}}{1\,\text{tab}}\right) \quad \text{Convert tab to mg, put tab in denominator}$$

$$\left(\frac{3\,\text{Rx}}{1}\right)\left(\frac{60\,\text{tab}}{1\text{Rx}}\right)\left(\frac{4\,\text{mg}}{1\,\text{tab}}\right)\left(\frac{1\,\text{dose}}{12\,\text{mg}}\right) \quad \text{Convert mg to dose, put mg in denominator}$$

$$\left(\frac{3}{1}\right)\left(\frac{60}{1}\right)\left(\frac{4}{1}\right)\left(\frac{1\,\text{dose}}{12}\right) \quad \text{Divide out Rx, tab, and mg, multiply across}$$

$$\frac{720\,\text{dose}}{12} \quad \text{Divide}$$

$$60\,\text{doses} \quad \text{Our Solution}$$

Practice - Dimensional Analysis

Use dimensional analysis to convert the following:

1) 7 mi. to yards

2) 234 oz. to tons

3) 11.2 mg to grams

4) 1.35 km to centimeters

5) 9,800,000 mm (milimeters) to miles

6) 4.5 ft^2 to square yards

7) 435,000 m^2 to squre kilometers

8) 8 km^2 to square feet

9) 0.0065 km^3 to cubic meters

10) 14.62 in^3 to cubic centimeters

11) 5,500 cm^3 to cubic yards

12) 3.5 mph (miles per hour) to feet per second

13) 185 yd. per min. to miles per hour

14) 153 ft/s (feet per second) to miles per hour

15) 248 mph to meters per second

16) 186,000 mph to kilometers per year

17) 7.50 $\mathrm{T/yd}^2$ (tons per square yard) to pounds per square inch

18) 16 $\mathrm{ft/s}^2$ to kilometers per hour squared

Use dimensional analysis to solve the following:

19) On a recent trip, Jan traveled 260 miles using 8 gallons of gas. How many

miles per 1-gallon did she travel? How many yards per 1-ounce?

20) A chair lift at the Divide ski resort in Cold Springs, WY is 4806 feet long and takes 9 minutes. What is the average speed in miles per hour? How many feet per second does the lift travel?

21) A certain laser printer can print 12 pages per minute. Determine this printer's output in pages per day, and reams per month. (1 ream = 5000 pages)

22) An average human heart beats 60 times per minute. If an average person lives to the age of 75, how many times does the average heart beat in a lifetime?

23) Blood sugar levels are measured in miligrams of gluclose per deciliter of blood volume. If a person's blood sugar level measured 128 mg/dL, how much is this in grams per liter?

24) You are buying carpet to cover a room that measures 38 ft by 40 ft. The carpet cost $18 per square yard. How much will the carpet cost?

25) A car travels 14 miles in 15 minutes. How fast is it going in miles per hour? in meters per second?

26) A cargo container is 50 ft long, 10 ft wide, and 8 ft tall. Find its volume in cubic yards and cubic meters.

27) A local zoning ordinance says that a house's "footprint" (area of its ground floor) cannot occupy more than $\frac{1}{4}$ of the lot it is built on. Suppose you own a $\frac{1}{3}$-acre lot, what is teh maximum allowed footprint for your house in square feet? in square inches? (1 acre = 43560 ft^2)

28) Computer memory is measured in units of bytes, where one byte is enough memory to store one character (a letter in the alphabet or a number). How many typical pages of text can be stored on a 700-megabyte compact disc? Assume that one typical page of text contains 2000 characters. (1 megabyte = 1,000,000 bytes)

29) In April 1996, the Department of the Interior released a "spike flood" from the Glen Canyon Dam on the Colorado River. Its purpose was to restore the river and the habitants along its bank. The release from teh dam lasted a week at a rate of 25,800 cubic feet of water per second. About how much water was released during the 1-week flood?

30) The largest single rough diamond ever found, the Cullinan diamond, weighed 3106 carats; how much does the diamond weigh in miligrams? in pounds? (1 carat - 0.2 grams)

Answers - Reduce Rational Expressions

1) $\{-10\}$

2) $\{0,2\}$

3) $\{-\frac{5}{2}\}$

4) $\{0,-10\}$

5) $\{0\}$

6) $\{-\frac{10}{3}\}$

7) $\{-2\}$

8) $\{0,-\frac{1}{2}\}$

9) $\{-8,4\}$

10) $\{0,\frac{1}{7}\}$

11) $\frac{7x}{6}$

12) $\frac{3}{n}$

13) $\frac{3}{5a}$

14) $\frac{7}{8k}$

15) $\frac{4}{x}$

16) $\frac{9x}{2}$

17) $\frac{3m-4}{10}$

18) $\frac{10}{9n^2(9n+4)}$

19) $\frac{10}{2p+1}$

20) $\frac{1}{9}$

21) $\frac{1}{x+7}$

22) $\frac{7m+3}{9}$

23) $\frac{8x}{7(x+1)}$

24) $\frac{7r+8}{8r}$

25) $\frac{n+6}{n+5}$

26) $\frac{b+6}{b+7}$

27) $\frac{9}{v-10}$

28) $\frac{3(x-3)}{5x+4}$

29) $\frac{2x-7}{5x-7}$

30) $\frac{k-4}{k+8}$

31) $\frac{3a-5}{5a+2}$

32) $\frac{9}{p+2}$

33) $\frac{2n-1}{9}$

34) $\frac{3x-5}{5(x+2)}$

35) $\frac{2(m+2)}{5m-3}$

36) $\frac{9r}{5(r+1)}$

37) $\frac{2(x-4)}{3x-4}$

38) $\frac{5b-8}{5b+2}$

39) $\frac{7n-4}{4}$

40) $\frac{5(v+1)}{3v+1}$

41) $\frac{(n-1)^2}{6(n+1)}$

42) $\frac{7x-6}{(3x+4)(x+1)}$

43) $\frac{7a+9}{2(3a-2)}$

44) $\frac{2(2k+1)}{9(k-1)}$

Answers - Multipy and Divide Rational Expressions

1) $4x^2$

2) $\frac{14}{3}$

3) $\frac{63}{10n}$

4) $\frac{63}{10m}$

5) $\frac{3x^5}{2}$

6) $\frac{5p}{2}$

7) $5m$

8) $\frac{7}{10}$

9) $\frac{r-6}{r+10}$

10) $x+4$

11) $\frac{2}{3}$

12) $\frac{9}{b-5}$

13) $\frac{x-10}{7}$

14) $\frac{1}{v-10}$

15) $x+1$

16) $\frac{a+10}{a-6}$

17) 5

18) $\frac{p-10}{p-4}$

19) $\frac{3}{5}$

20) $\frac{x+10}{x+4}$

21) $\frac{4(m-5)}{5m^2}$

22) 7

23) $\frac{x+3}{4}$

24) $\frac{n-9}{n+7}$

25) $\frac{b+2}{8b}$

26) $\frac{v-9}{5}$

27) $-\frac{1}{n-6}$

28) $\frac{x+1}{x-3}$

29) $\frac{1}{a+7}$

30) $\frac{7}{8(k+3)}$

31) $\frac{x-4}{x+3}$

32) $\frac{9(x+6)}{10}$

33) $9m^2(m+10)$

34) $\frac{10}{9(n+6)}$

35) $\frac{p+3}{6(p+8)}$

36) $\frac{x-8}{x+7}$

37) $\frac{5b}{b+5}$

38) $n+3$

39) $r-8$

40) $\frac{18}{5}$

Answers - Least Common Denominators

1) 18

2) a^2

3) ay

4) $20xy$

5) $6a^3c^3$

6) 12

7) $2x-8$

8) x^2-2x-3

9) x^2-x-12

10) $x^2-11x+30$

11) $12a^4b^5$

12) $25x^3y^5z$

13) $x(x-3)$

14) $4(x-2)$

15) $(x+2)(x-4)$

16) $x(x-7)(x+1)$

17) $(x+5)(x-5)$

18) $(x-3)^2(x+3)$

19) $(x+1)(x+2)(x+3)$

20) $(x-2)(x-5)(x+3)$

21) $\frac{6a^4}{10a^3b^2}, \frac{2b}{10a^3b}$

22) $\frac{3x^2+6x}{(x-4)(x+2)}, \frac{2x-8}{(x-4)(x+2)}$

23) $\frac{x^2+4x+4}{(x-3)(x+2)}, \frac{2x^2-8x}{(x-4)(x+3)(x+1)}$

24) $\frac{5}{x(x-6)}, \frac{2x-12}{x(x-6)}, \frac{-3x}{x(x-6)}$

25) $\frac{x^2-4x}{(x-4)^2(x+4)}, \frac{3x^2+12x}{(x-4)^2(x+4)}$

26) $\frac{5x+1}{(x-5)(x+2)}, \frac{4x+8}{(x-5)(x+2)}$

27) $\frac{x^2+7x+6}{(x-6)(x+6)^2}, \frac{2x^2-9x-18}{(x-6)(x+6)^2}$

28) $\frac{3x^2+4x+1}{(x-4)(x+3)(x+1)}, \frac{2x^2-8x}{(x-4)(x+3)(x+1)}$

29) $\frac{4x}{(x-3)(x+2)}, \frac{x^2+4x+4}{(x-3)(x+2)}$

30) $\frac{3x^2+15x}{(x-4)(x-2)(x+5)}, \frac{x^2-4x+4}{(x-4)(x-2)(x+5)}, \frac{5x-20}{(x-4)(x-2)(x+5)}$

Answers - Adding Rational Expressions

1) $\frac{6}{a+3}$

2) $x-4$

3) $t+7$

4) $\frac{a+4}{a+6}$

5) $\frac{x+6}{x-5}$

6) $\frac{3x+4}{x^2}$

7) $\frac{5}{24r}$

8) $\frac{7x+3y}{x^2y^2}$

9) $\frac{16-15t}{18t^3}$

10) $\frac{5x+9}{24}$

11) $\frac{a+8}{4}$

12) $\frac{5a^2+7a-3}{9a^2}$

13) $\frac{-7x-13}{4x}$

14) $\frac{c^2+3cd-d^2}{c^2d^2}$

15) $\frac{3y^2-3xy-6x^2}{2x^2y^2}$

16) $\frac{4x}{x^2-1}$

17) $\frac{-z^2+5z}{z^2-1}$

18) $\frac{11x+15}{4x(x+5)}$

19) $\frac{14-3x}{x^2-4}$

20) $\frac{x^2-x}{x^2-25}$

21) $\frac{4t-5}{4(t-3)}$

22) $\frac{2x+10}{(x+3)^2}$

23) $\frac{6-20x}{15x(x+1)}$

24) $\frac{9a}{4(a-5)}$

25) $\frac{5t^2+2ty-y^2}{y^2-t^2}$

26) $\frac{2t^2-10t+25}{x(x-5)}$

27) $\frac{x-3}{(x+1)(x+4)}$

28) $\frac{2x+3}{(x-1)(x+4)}$

29) $\frac{x-8}{(x+8)(x+6)}$

30) $\frac{2x-5}{(x-3)(x-2)}$

31) $\frac{5x+12}{x^2+5x+6}$

32) $\frac{4x+1}{x^2-x-3}$

33) $\frac{2x+4}{x^2+4x+3}$

34) $\frac{2x+7}{x^2+5x+6}$

35) $\frac{2x-8}{x^2-5x-14}$

36) $\frac{-3x^2+7x+4}{3(x+2)(2-x)}$

37) $\frac{a-2}{a^2-9}$

38) $\frac{2}{y^2-y}$

39) $\frac{z-3}{2z-1}$

40) $\frac{2}{r+s}$

41) $\frac{5x-5}{x^2-5x-14}$

42) $\frac{5x+5}{x^2+2x-15}$

43) $\frac{29-x}{x^2+2x-14}$

44) $\frac{5x-10}{x^2+5x+4}$

Answers - Complex Fractions

1) $\frac{x}{x-1}$

2) $\frac{1-y}{y}$

3) $\frac{-a}{a+2}$

4) $\frac{5-a}{a}$

5) $-\frac{a-1}{a+1}$

6) $\frac{b}{2(2-b)}$

7) $\frac{2}{5}$

8) $\frac{4}{5}$

9) $-\frac{1}{2}$

10) $-\frac{1}{2}$

11) $\frac{x^2-x-1}{x^2+x+1}$

12) $\frac{2a^2-3a+3}{3a-2}$

13) $\frac{x}{3}$

14) $3x+2$

15) $\frac{4b(a-b)}{a}$

16) $\frac{x+2}{x-1}$

17) $\frac{x-5}{x+9}$

18) $-\frac{(x-3)(x+5)}{4x^2-5x+4}$

19) $\frac{1}{3x+8}$

20) $\frac{1}{x+4}$

21) $\frac{x-2}{x+2}$

22) $\frac{x-7}{x+5}$

23) $\frac{x-3}{x+4}$

24) $-\frac{2(a+1)}{7a-4}$

25) $-\frac{b-2}{2b+3}$

26) $\frac{x+y}{x-y}$

27) $\frac{a-3b}{a+3b}$

28) $-\frac{2x}{x^2+1}$

29) $-\frac{2}{y}$

30) x^2-1

31) $\frac{y-x}{xy}$

32) $\frac{x^2-xy+y^2}{y-x}$

33) $\frac{x^2+y^2}{xy}$

34) $\frac{2x-1}{2x+1}$

35) $\frac{1-3x}{1+3x}$

36) $\frac{x+y}{xy}$

Answers - Proportions

1) $\frac{40}{3}=a$

2) $n=\frac{14}{3}$

3) $k=\frac{12}{7}$

4) $x=16$

5) $x=\frac{3}{2}$

6) $n=34$

7) $m=\frac{17}{7}$

8) $x=\frac{79}{8}$

9) $p=49$

10) $n=25$

11) $b=-\frac{40}{3}$

12) $r=\frac{36}{5}$

13) $x=\frac{5}{2}$

14) $n=\frac{32}{5}$

15) $a=\frac{6}{7}$

16) $v=-\frac{16}{7}$

17) $v=\frac{69}{5}$

18) $n=\frac{61}{3}$

19) $x=\frac{38}{3}$

20) $k=\frac{73}{3}$

21) $x=-8,5$

22) $x=-7,5$

23) $m=-7,8$

24) $x=-3,9$

25) $p=-7,-2$

26) $n=-6,9$

27) $n=-1$

28) $n=-4,-1$

29) $x=-7,1$

30) $x=-1,3$

31) \$9.31

32) 16

33) 2.5 in

34) 12.1 ft

35) 39.4 ft

36) 3.1 in

37) T: 38, V: 57

38) J: 4 hr, S: 14 hr

39) $8

40) C: 36 min,
K: 51 min

Answers - Rational Equations

1) $-\frac{1}{2}, \frac{2}{3}$

2) $-3, 1$

3) 3

4) $-1, 4$

5) 2

6) $\frac{1}{3}$

7) -1

8) $-\frac{1}{3}$

9) -5

10) $-\frac{7}{15}$

11) $-5, 0$

12) $5, 10$

13) $\frac{16}{3}, 5$

14) 2, 13

15) -8

16) 2

17) $-\frac{1}{5}, 5$

18) $-\frac{9}{5}, 1$

19) $\frac{3}{2}$

20) 10

21) 0, 5

22) $-2, \frac{5}{3}$

23) $4, 7$

24) -1

25) $\frac{2}{3}$

26) $\frac{1}{2}$

27) $\frac{3}{10}$

28) 1

29) $-\frac{2}{3}$

30) -1

31) 1

32) $\frac{13}{4}$

33) -10

34) $\frac{7}{4}$

Answers - Dimensional Analysis

1) 12320 yd

2) 0.0073125 T

3) 0.0112 g

4) 135,000 cm

5) 6.1 mi

6) 0.5 yd^2

7) 0.435 km^2

8) 86,067,200 ft^2

9) 6,500,000 m^3

10) 2.3958 cm^3

11) 0.0072 yd^3

12) 5.13 ft/sec

13) 6.31 mph

14) 104.32 mi/hr

15) 111 m/s

16) 2,623,269,600 km/yr

17) 11.6 lb/in^2

18) 63,219.51 km/hr^2

19) 32.5 mph; 447 yd/oz

20) 6.608 mi/hr

21)17280 pages/day; 103.4 reams/month

22) 2,365,200,000 beats/lifetime

23) 1.28 g/L

24) $3040

25) 56 mph; 25 m/s

26) 148.15 yd^3

27) 3630 ft^2

28) 350,000 pages

29) 15,603,840,000 ft^3/week

30) 621,200 mg; 1368 lb