Beginning and Intermediate Algebra

Chapter 5: Polynomials

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Chapter 5: Polynomials

5.1

Polynomials - Exponent Properties

Problems with expoenents can often be simplified using a few basic exponent properties. Exponents represent repeated multiplication. We will use this fact to discover the important properties.

Example 1.

$$\begin{array}{rl} a^3a^2 & \text{Expand exponents to multiplication problem} \\ (aaa)(aa) & \text{Now we have } 5\, a's \text{ being multiplied together} \\ a^5 & \text{Our Solution} \end{array}$$

A quicker method to arrive at our answer would have been to just add the exponents: $a^3a^2 = a^{3+2} = a^5$ This is known as the **product rule of exponents**

Product Rule of Exponents: $a^m a^n = a^{m+n}$

The product rule of exponents can be used to simplify many problems. We will add the exponent on like variables. This is shown in the following examples

Example 2.

$$\begin{array}{rl} 3^2 \cdot 3^6 \cdot 3 & \text{Same base, add the exponents } 2+6+1 \\ 3^9 & \text{Our Solution} \end{array}$$

Example 3.

$$\begin{array}{rl} 2x^3y^5z \cdot 5xy^2z^3 & \text{Multiply } 2 \cdot 5, \text{ add exponents on } x, y \text{ and } z \\ 10x^4y^7z^4 & \text{Our Solution} \end{array}$$

Rather than multiplying, we will now try to divide with exponents

Example 4.

$$\begin{array}{rl} \dfrac{a^5}{a^2} & \text{Expand exponents} \\ \dfrac{aaaaa}{aa} & \text{Divide out two of the } a's \\ aaa & \text{Convert to exponents} \\ a^3 & \text{Our Solution} \end{array}$$

A quicker method to arrive at the solution would have been to just subtract the exponents, $\frac{a^5}{a^2} = a^{5-2} = a^3$. This is known as the quotient rule of exponents.

$$\textbf{Quotient Rule of Exponents: } \boldsymbol{\frac{a^m}{a^n} = a^{m-n}}$$

The quotient rule of exponents can similarly be used to simplify exponent problems by subtracting exponents on like variables. This is shown in the following examples.

Example 5.

$$\begin{array}{rl} \dfrac{7^{13}}{7^5} & \text{Same base, subtract the exponents} \\ 7^8 & \text{Our Solution} \end{array}$$

Example 6.

$$\begin{array}{rl} \dfrac{5a^3b^5c^2}{2\mathrm{ab}^3c} & \text{Subtract exponents on } a, b \text{ and } c \\ \\ \dfrac{5}{2}a^2b^2c & \text{Our Solution} \end{array}$$

A third property we will look at will have an exponent problem raised to a second exponent. This is investigated in the following example.

Example 7.

$$\begin{array}{rl} \left(a^2\right)^3 & \text{This means we have } a^2 \text{ three times} \\ a^2 \cdot a^2 \cdot a^2 & \text{Add exponents} \\ a^6 & \text{Our solution} \end{array}$$

A quicker method to arrive at the solution would have been to just multiply the exponents, $(a^2)^3 = a^{2\cdot 3} = a^6$. This is known as the power of a power rule of exponents.

$$\textbf{Power of } \boldsymbol{a} \textbf{ Power Rule of Exponents: } \boldsymbol{(a^m)^n = a^{mn}}$$

This property is often combined with two other properties which we will investigate now.

Example 8.

$$\begin{array}{rl} (ab)^3 & \text{This means we have } (ab) \text{ three times} \\ (ab)(ab)(ab) & \text{Three } a's \text{ and three } b's \text{ can be written with exponents} \\ a^3b^3 & \text{Our Solution} \end{array}$$

A quicker method to arrive at the solution would have been to take the exponent of three and put it on each factor in parenthesis, $(ab)^3 = a^3b^3$. This is known as the power of a product rule or exponents.

$$\textbf{Power of } \boldsymbol{a} \textbf{ Product Rule of Exponents: } (\mathbf{ab})^{\boldsymbol{m}} = \boldsymbol{a^m b^m}$$

It is important to careful to only use the power of a product rule with multiplication inside parenthesis. This property does NOT work if there is addition or subtraction.

Warning 9.

$$(a+b)^m \neq a^m + b^m \qquad \text{These are } \textbf{NOT} \text{ equal, beware of this error!}$$

Another property that is very similar to the power of a product rule is considered next.

Example 10.

$$\left(\frac{a}{b}\right)^3 \qquad \text{This means we have the fraction three timse}$$

$$\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) \qquad \text{Multiply fractions across top and bottom, using exponents}$$

$$\frac{a^3}{b^3} \qquad \text{Our Solution}$$

A quicker method to arrive at the solution would have been to put the exponent on every factor in both the numerator and denominator, $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$. This is known as the power of a quotient rule of exponents.

$$\textbf{Power of } \boldsymbol{a} \textbf{ Quotient Rule of Exponents: } \left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\right)^{\boldsymbol{m}} = \frac{\boldsymbol{a^m}}{\boldsymbol{b^m}}$$

The power of a power, product and quotient rules are often used together to simplify expressions. This is shown in the following examples.

Example 11.

$$(x^3yz^2)^4 \qquad \text{Put the exponent of 4 on each factor, multiplying powers}$$

$$x^{12}y^4z^8 \qquad \text{Our solution}$$

Example 12.

$$\left(\frac{a^3b}{c^4d^5}\right)^2 \quad \text{Put the exponent of 2 on each factor, multiplying powers}$$

$$\frac{a^6b^2}{c^4d^{10}} \quad \text{Our Solution}$$

As we multiply exponents its important to remember these properties apply to exponents, not bases. An expressions such as 5^3 does not mean we multipy 5 by 3, rather we multiply 5 three times, $5 \times 5 \times 5 = 125$. This is shown in the next example.

Example 13.

$$(4x^2y^5)^3 \quad \text{Put the exponent of 3 on each factor, multiplying powers}$$

$$4^3x^6y^{15} \quad \text{Evaluate } 4^3$$

$$64x^6y^{15} \quad \text{Our Solution}$$

In the previous example we did not put the 3 on the 4 and multipy to get 12, this would have been incorrect. Never multipy a base by the exponent. These properties pertain to exponents only, not bases.

In this lesson we have discussed 5 different exponent properties. These rules are summarized in the following table.

Rules of Exponents

Product Rule of Exponents	$a^ma^n = a^{m+n}$
Quotient Rule of Exponents	$\dfrac{a^m}{a^n} = a^{m-n}$
Power of *a* Power Rule of Exponents	$(a^m)^n = a^{mn}$
Power of *a* Product Rule of Exponents	$(ab)^m = a^mb^m$
Power of *a* Quotient Rule of Exponents	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

These five properties are often mixed up in the same problem. Often there is a bit of flexibility as to which property is used first. However, order of operations still applies to a problem. For this reason it is the suggestion of the auther to simplify inside any parenthesis first, then simplify any exponents (using power rules), and finally simplify any multiplication or division (using product and quotent rules). This is illustrated in the next few examples.

Example 14.

$$(4x^3y \cdot 5x^4y^2)^3 \quad \text{In parenthesis simplify using product rule, adding exponents}$$

$$(20x^7y^3)^3 \quad \text{With power rules, put three on each factor, multiplying exponents}$$

$$20^3x^{21}y^9 \quad \text{Evaluate } 20^3$$

$$8000x^{21}y^9 \quad \text{Our Solution}$$

Example 15.

$$\begin{array}{rl} 7a^3(2a^4)^3 & \text{Parenthesis are already simplified, next use power rules} \\ 7a^3(8a^{12}) & \text{Using product rule, add exponents and multiply numbers} \\ 56a^{15} & \text{Our Solution} \end{array}$$

Example 16.

$$\begin{array}{rl} \dfrac{3a^3b\cdot 10a^4b^3}{2a^4b^2} & \text{Simplify numerator with product rule, adding exponents} \\ \\ \dfrac{30a^7b^4}{2a^4b^2} & \text{Now use the quotient rule to subtract exponents} \\ \\ 15a^3b^2 & \text{Our Solution} \end{array}$$

Example 17.

$$\begin{array}{rl} \dfrac{3m^8n^{12}}{(m^2n^3)^3} & \text{Use power rule in denominator} \\ \\ \dfrac{3m^8n^{12}}{m^6n^9} & \text{Use quotient rule} \\ \\ 3m^2n^3 & \text{Our solution} \end{array}$$

Example 18.

$$\begin{array}{rl} \left(\dfrac{3\mathrm{ab}^2(2a^4b^2)^3}{6a^5b^7}\right)^2 & \text{Simplify inside parenthesis first, using power rule in numerator} \\ \\ \left(\dfrac{3\mathrm{ab}^2(8a^{12}b^6)}{6a^5b^7}\right)^2 & \text{Simplify numerator using product rule} \\ \\ \left(\dfrac{24a^{13}b^8}{6a^5b^7}\right)^2 & \text{Simplify using the quotient rule} \\ \\ \left(4a^8b\right)^2 & \text{Now that the parenthesis are simplified, use the power rules} \\ 16a^{16}b^2 & \text{Our Solution} \end{array}$$

Clearly these problems can quickly become quite involved. Remember to follow order of operations as a guide, simplify inside parenthesis first, then power rules, then product and quotient rules.

Practice - Exponent Properties

Simplify.

1) $4 \cdot 4^4 \cdot 4^4$

2) $4 \cdot 4^4 \cdot 4^2$

3) $4 \cdot 2^2$

4) $3 \cdot 3^3 \cdot 3^2$

5) $3m \cdot 4\mathrm{mn}$

6) $3x \cdot 4x^2$

7) $2m^4n^2 \cdot 4\mathrm{nm}^2$

8) $x^2 y^4 \cdot \mathrm{xy}^2$

9) $(3^3)^4$

10) $(4^3)^4$

11) $(4^4)^2$

12) $(3^2)^3$

13) $(2u^3v^2)^2$

14) $(\mathrm{xy})^3$

15) $(2a^4)^4$

16) $(2\mathrm{xy})^4$

17) $\frac{4}{4^3}$

18) $\frac{3}{3^3}$

19) $\frac{3^2}{3}$

20) $\frac{3^4}{3}$

21) $\frac{3\mathrm{nm}^2}{3n}$

22) $\frac{x^2y^4}{4\mathrm{xy}}$

23) $\frac{4x^3y^3}{3\mathrm{xy}^4}$

24) $\frac{\mathrm{xy}^3}{4\mathrm{xy}}$

25) $(x^3y^4 \cdot 2x^2y^3)^2$

26) $(u^2v^2 \cdot 2u^4)^3$

27) $2x(x^4y^4)^4$

29) $\frac{2x^3y^2}{3x^3y^4 \cdot 4x^2y^3}$

28) $\frac{3\mathrm{vu}^{4} \cdot 2v^2}{u^4v^2 \cdot 2u^3v^4}$

31) $(\frac{(2x)^3}{x^3})^2$

30) $\frac{2ba^2 \cdot 2b^4}{ba^2 \cdot 3a^3b^4}$

33) $(\frac{2y}{(2x^2y^4)^4})^3$

32) $\frac{2a^2b^2}{(ba^4)^2}$

35) $(\frac{2mn^4}{mn^4 \cdot 2m^4n^4})^3$

34) $\frac{yx^2 \cdot (y^4)^2}{2y^4}$

37) $\frac{2xy^3 \cdot 2x^2y^2}{2xy^4 \cdot y^3}$

36) $\frac{n^3(n^4)^2}{2mn}$

39) $\frac{q^3r^2 \cdot (2p^2q^2r^3)^2}{2p^3}$

38) $\frac{(2yx^2)^2}{2x^2y^4 \cdot x^2}$

41) $(\frac{zy^3 \cdot zx^2y^4}{x^3y^3z^3})^4$

40) $\frac{2x^4y^3 \cdot 2zx^2y^3}{(xy^2z^2)^4}$

43) $\frac{2x^2y^2z^2 \cdot 2zx^2y^2}{(x^2z^3)^2}$

42) $(\frac{2qp^3r^4 \cdot 2p^3}{(qrp^3)^2})^4$

5.2

Polynomials - Negative Exponents

There are a few special exponent properties that deal with exponents that are not positive. The first is considered in the following example, which is worded out 2 different ways:

Example 19.

$\frac{a^3}{a^3}$	Use the quotient rule to subtract exponents
a^0	Our Solution, but now we consider the problem a second way:
$\frac{a^3}{a^3}$	Rewrite exponents as repeated multiplication
$\frac{aaa}{aaa}$	Reduce out all the $a's$
$\frac{1}{1}=1$	Our Solution, when we combine the two solutions we get:
$a^0=1$	Our final result.

This final result is an imporant property known as the zero power rule of exponents

Zero Power Rule of Exponents: $a^0=1$

Any number or expression raised to the zero power will always be 1. This is illustrated in the following example.

Example 20.

$(3x^2)^0$	Zero power rule
1	Our Solution

Another property we will consider here deals with negative exponents. Again we will solve the following example two ways.

Example 21.

$$\frac{a^3}{a^5} \quad \text{Using the quotient rule, subtract exponents}$$

$$a^{-2} \quad \text{Our Solution, but we will also solve this problem another way.}$$

$$\frac{a^3}{a^5} \quad \text{Rewrite exponents as repeated multiplication}$$

$$\frac{aaa}{aaaaa} \quad \text{Reduce three } a's \text{ out of top and bottom}$$

$$\frac{1}{aa} \quad \text{Simplify to exponents}$$

$$\frac{1}{a^2} \quad \text{Our Solution, putting these solutions together gives:}$$

$$a^{-2} = \frac{1}{a^2} \quad \text{Our Final Solution}$$

This example illustrates an important property of exponents. Negative exponents yeild the reciprocal of the base. Once we take the reciprical the exponent is now positive. Also, it is important to note a negative exponent does not mean the expression is negative, only that we need the reciprocal of the base. Following are the rules of negative exponents

Rules of Negative Exponets:

$$\boldsymbol{a^{-m} = \frac{1}{m}}$$

$$\boldsymbol{\frac{1}{a^{-m}} = a^m}$$

$$\boldsymbol{\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}}$$

Negative exponents can be combined in several different ways. As a general rule if we think of our expression as a fraction, negative exponents in the numerator must be moved to the denominator, likewise, negative exponents in the denominator need to be moved to the numerator. When the base with exponent moves, the exponent is now positive. This is illustrated in the following example.

Example 22.

$$\frac{a^3b^{-2}c}{2d^{-1}e^{-4}f^2} \quad \text{Negative exponents on } b, d, \text{ and } e \text{ need to flip}$$

$$\frac{a^3cde^4}{2b^2f^2} \quad \text{Our Solution}$$

As we simplified our fraction we took special care to move the bases that had a negative exponent, but the expression itself did not become negative because of those exponents. Also, it is important to remember that exponents only effect what they are attached to. The 2 in the denominator of the above example does not have an exponent on it, so it does not move with the d.

We now have the following nine properties of exponents. It is important that we are very familiar with all of them.

Properties of Exponents

$$\boldsymbol{a^m a^n = a^{m+n}} \qquad \boldsymbol{(ab)^m = a^m b^m} \qquad \boldsymbol{a^{-m} = \frac{1}{a^m}}$$

$$\boldsymbol{\frac{a^m}{a^n} = a^{m-n}} \qquad \boldsymbol{\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}} \qquad \boldsymbol{\frac{1}{a^{-m}} = a^m}$$

$$\boldsymbol{(a^m)^n = a^{mn}} \qquad \boldsymbol{a^0 = 1} \qquad \boldsymbol{\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}}$$

Simplifying with negative exponents is much the same as simplifying with positive exponents. It is the advice of the author to keep the negative exponents until the end of the problem and then move them around to their correct location (numerator or denominator). As we do this it is important to be very careful of rules for adding, subtracting, and multiplying with negatives. This is illustrated in the following examples

Example 23.

$$\frac{4x^{-5}y^{-3}\cdot 3x^3y^{-2}}{6x^{-5}y^3} \quad \text{Simplify numerator with product rule, adding exponents}$$

$$\frac{12x^{-2}y^{-5}}{6x^{-5}y^3} \quad \text{Quotient rule to subtract exponets, be careful with negatives!}$$

$$(-2)-(-5)=(-2)+5=3$$

$$(-5)-3=(-5)+(-3)=-8$$

$$2x^3y^{-8} \quad \text{Negative exponent needs to move down to denominator}$$

$$\frac{2x^3}{y^8} \quad \text{Our Solution}$$

Example 24.

$$\frac{(3\mathrm{ab}^3)^{-2}\mathrm{ab}^{-3}}{2a^{-4}b^0}$$ In numerator, use power rule with -2, multiplying exponnets
In denominator, $b^0=1$

$$\frac{3^{-2}a^{-2}b^{-2}\mathrm{ab}^{-3}}{2a^{-4}}$$ In numerator, use product rule to add exponents

$$\frac{3^{-2}a^{-1}b^{-5}}{2a^{-4}}$$ Use quotient rule to subtract exponents, be careful with negatives

$(-1)-(-4)=(-1)+4=3$

$$\frac{3^{-2}a^3b^{-5}}{2}$$ Move 3 and b to denominator because of negative exponents

$$\frac{a^3}{3^2 2b^5}$$ Evaluate $3^2 2$

$$\frac{a^3}{18b^5}$$ Our Solution

In the previous example it is important to point out that when we simplified 3^{-2} we moved the three to the denominator and the exponent became positive. We did not make the number negative! Negative exponents never make the bases negative, they simply mean we have to take the reciprocal of the base. One final example with negative exponents is given here.

Example 25.

$$\left(\frac{3x^{-2}y^5z^3\cdot 6x^{-6}y^{-2}z^{-3}}{(9x^2y^{-2})^{-3}}\right)^{-3}$$ In numerator, use product rule, adding exponents
In denominator, use power rule, multiplying exponets

$$\left(\frac{18x^{-8}y^3z^0}{9^{-3}x^{-6}y^6}\right)^{-3}$$ Use quotient rule to subtract exponents,

be careful with negatives:

$(-8)-(-6)=(-8)+6=-2$

$3-6=3+(-6)=-3$

$(2x^{-2}y^{-3}z^0)^{-3}$ Parenthesis are done, use power rule with -3

$2^{-3}x^6y^9z^0$ Move 2 with negative exponent down and $z^0=1$

$$\frac{x^6y^9}{2^3}$$ Evaluate 2^3

$$\frac{x^6y^9}{8}$$ Our Solution

Practice - Negative Exponents

Simplify. Your answer should contain only positive expontents.

1) $2x^4y^{-2}\cdot(2xy^3)^4$

2) $2a^{-2}b^{-3}\cdot(2a^0b^4)^4$

3) $(a^4b^{-3})^3\cdot 2a^3b^{-2}$

4) $2x^3y^2\cdot(2x^3)^0$

5) $(2x^2y^2)^4x^{-4}$

6) $(m^0n^3\cdot 2m^{-3}n^{-3})^0$

7) $(x^3y^4)^3\cdot x^{-4}y^4$

8) $2m^{-1}n^{-3}\cdot(2m^{-1}n^{-3})^4$

9) $\frac{2x^{-3}y^2}{3x^{-3}y^3\cdot 3x^0}$

10) $\frac{3y^3}{3yx^3\cdot 2x^4y^{-3}}$

11) $\frac{4xy^{-3}\cdot x^{-4}y^0}{4y^{-1}}$

12) $\frac{3x^3y^2}{4y^{-2}\cdot 3x^{-2}y^{-4}}$

13) $\frac{u^2v^{-1}}{2u^0v^4\cdot 2uv}$

14) $\frac{2xy^2\cdot 4x^3y^{-4}}{4x^{-4}y^{-4}\cdot 4x}$

15) $\frac{u^2}{4u^0v^3\cdot 3v^2}$

16) $\frac{2x^{-2}y^2}{4yx^2}$

17) $\frac{2y}{(x^0y^2)^4}$

18) $\frac{(a^4)^4}{2b}$

19) $(\frac{2a^2b^3}{a^{-1}})^4$

20) $(\frac{2y^{-4}}{x^2})^{-2}$

21) $\frac{2nm^4}{(2m^2n^2)^4}$

22) $\frac{2y^2}{(x^4y^0)^{-4}}$

23) $\frac{(2mn)^4}{m^0n^{-2}}$

24) $\frac{2x^{-3}}{(x^4y^{-3})^{-1}}$

25) $\frac{y^3\cdot x^{-3}y^2}{(x^4y^2)^3}$

26) $\frac{2x^{-2}y^0\cdot 2xy^4}{(xy^0)^{-1}}$

27) $\frac{2u^{-2}v^3\cdot(2uv^4)^{-1}}{2u^{-4}v^0}$

28) $\frac{2yx^2\cdot x^{-2}}{(2x^0y^4)^{-1}}$

29) $(\frac{2x^0\cdot y^4}{y^4})^3$

30) $\frac{u^{-3}v^{-4}}{2v(2u^{-3}v^4)^0}$

31) $\frac{y(2x^4y^2)^2}{2x^4y^0}$

32) $\frac{b^{-1}}{(2a^4b^0)^0\cdot 2a^{-3}b^2}$

33) $\frac{2yzx^2}{2x^4y^4z^{-2}\cdot(zy^2)^4}$

34) $\frac{2b^4c^{-2}\cdot(2b^3c^2)^{-4}}{a^{-2}b^4}$

35) $\frac{2kh^0\cdot 2h^{-3}k^0}{(2kj^3)^2}$

36) $(\frac{(2x^{-3}y^0z^{-1})^3\cdot x^{-3}y^2}{2x^3})^{-2}$

37) $\frac{(cb^3)^2\cdot 2a^{-3}b^2}{(a^3b^{-2}c^3)^3}$

38) $\frac{2q^4\cdot m^2p^2q^4}{(2m^{-4}p^2)^3}$

39) $\frac{(yx^{-4}z^2)^{-1}}{z^3\cdot x^2y^3z^{-1}}$

40) $\frac{2mpn^{-3}}{(m^0n^{-4}p^2)^3\cdot 2n^2p^0}$

5.3

Polynomials - Scientific Notation

One application of exponent properties comes from scientific notation. Scientific notation is used to represent really large or really small numbers. An example of really large numbers would be the distance that light travels in a year in miles. An example of really small numbes would be the mass of a single hydrogen atom in grams. Doing basic operations such as multiplication and division with these numbers would normally be very combersome. However, our exponent properties make this process much simpler.

First we will take a look at what scientific notation is. Scientific notation has two parts, a number between one and ten (it can be equal to one, but not ten), and that number multiplied by ten to some exponent.

$$\textbf{Scientific Notation: } \boldsymbol{a \times 10^b} \textbf{ where } \boldsymbol{1 \leqslant a < 10}$$

The exponent, b, is very important to how we convert between scientific notation and normal numbers, or standard notation. The exponent tells us how many times we will multiply by 10. Multiplying by 10 in effect moves the decimal point one place. So the exponent will tell us how many times the exponent moves between scientific notation and standard notation. To decide which direction to move the decimal (left or right) we simply need to remember that positive exponents mean in standard notation we have a big number (bigger than ten) and negative exponents mean in standard notation we have a small number (less than one).

Keeping this in mind, we can easily make conversions between standard notation and scientific notation.

Example 26.

Convert $14,200$ to scientific notation	Put decimal after first nonzero number
1.42	Exponent is how many times decimal moved, 4
$\times 10^4$	Positive exponent, standard notation is big
1.42×10^4	Our Solution

Example 27.

Convert 0.0042 to scientific notation	Put decimal after first nonzero number
4.2	Exponent is how many times decimal moved, 3
$\times 10^{-3}$	Negative exponent, standard notation is small
4.2×10^{-3}	Our Solution

Example 28.

Convert 3.21×10^5 to standard notation	Positive exponent means standard notation big number. Move decimal right 5 places
$321,000$	Our Solution

Example 29.

Conver 7.4×10^{-3} to standard notation	Negative exponent means standard notation is a small number. Move decimal left 3 places
0.0074	Our Solution

Converting between standard notation and scientific notation is important to understand how scientific notation works and what it does. Here our main interest is to be able to multiply and divide number in scientific notation using exponent properties. The way we do this is first do the operation with the front number (multiply or divide) then use exponent properties to simplify the 10's. Scientific notation is the only time where it will be allowed to have negative exponents in our final solution. The negative exponent simply informs us that we are dealing with small numbers. Consider the following examples.

Example 30.

$(2.1\times 10^{-7})(3.7\times 10^5)$	Deal with numbers and $10's$ separately
$(2.1)(3.7)=7.77$	Multiply numbers
$10^{-7}10^5=10^{-2}$	Use product rule on $10's$ and add exponents
7.77×10^{-2}	Our Solution

Example 31.

$\dfrac{4.96\times 10^4}{3.1\times 10^{-3}}$	Deal with numbers and $10's$ separately
$\dfrac{4.96}{3.1}=1.6$	Divide Numbers
$\dfrac{10^4}{10^{-3}}=10^7$	Use quotient rule to subtract exponents, be careful with negatives!

$$\begin{array}{rl} & \text{Becarful with negatives, } 4-(-3)=4+3=7 \\ 1.6\times 10^{7} & \text{Our Solution} \end{array}$$

Example 32.

$$\begin{array}{rl} (1.8\times 10^{-4})^{3} & \text{Use power rule to deal with numbers and } 10's \text{ separately} \\ 1.8^{3}=5.832 & \text{Evaluate } 1.8^{3} \\ (10^{-4})^{3}=10^{-12} & \text{Multiply exponents} \\ 5.832\times 10^{-12} & \text{Our Solution} \end{array}$$

Often when we multiply or divide in scientific notation the end result is not in scientific notation. We will then have to convert the front number into scientific notation and then combine the 10's using the product property of expoents and adding the exponents. This is shown in the following examples

Example 33.

$$\begin{array}{rl} (4.7\times 10^{-3})(6.1\times 10^{9}) & \text{Deal with numbers and } 10's \text{ separately} \\ (4.7)(6.1)=28.67 & \text{Multiply numbers} \\ 2.867\times 10^{1} & \text{Convert this number into scientific notation} \\ \mathbf{10^{1}}10^{-3}10^{9}=10^{7} & \text{Use product rule, add exponents, using } 10^{1} \text{ from conversion} \\ 2.867\times 10^{7} & \text{Our Solution} \end{array}$$

Example 34.

$$\begin{array}{rl} \dfrac{2.014\times 10^{-3}}{3.8\times 10^{-7}} & \text{Deal with numbers and } 10's \text{ separately} \\ \dfrac{2.014}{3.8}=0.53 & \text{Divide numbers} \\ 0.53=5.3\times 10^{-1} & \text{Change this number into scientific notation} \\ \dfrac{\mathbf{10^{-1}}10^{-3}}{10^{-7}}=10^{3} & \text{Use product and quotient rule, using } 10^{-1} \text{ from the conversion} \\ & \text{Be careful with signs:} \\ & (-1)+(-3)-(-7)=(-1)+(-3)+7=3 \\ 5.3\times 10^{3} & \text{Our Solution} \end{array}$$

Practice - Scientific Notation

Write each number in scientific notiation

1) 885

2) 0.000744

3) 0.081

4) 1.09

5) 0.039

6) 15000

Write each number in standard notation

7) $8.7 \text{ x } 10^{5}$

8) $2.56 \text{ x } 10^{2}$

9) $9 \text{ x } 10^{-4}$

10) $5 \text{ x } 10^{4}$

11) $2 \text{ x } 10^{0}$

12) $6 \text{ x } 10^{-5}$

Simplify. Write each answer in scientific notation.

13) $(7 \text{ x } 10^{-1})(2 \text{ x } 10^{-3})$

14) $(2 \times 10^{-6})(8.8 \times 10^{-5})$

15) $(5.26 \text{ x } 10^{-5})(3.16 \text{ x } 10^{-2})$

16) $(5.1 \times 10^{6})(9.84 \times 10^{-1})$

17) $(2.6 \text{ x } 10^{-2})(6 \text{ x } 10^{-2})$

18) $\frac{7.4 \times 10^{4}}{1.7 \times 10^{-4}}$

19) $\frac{4.9 \times 10^{1}}{2.7 \times 10^{-3}}$

20) $\frac{7.2 \times 10^{-1}}{7.32 \times 10^{-1}}$

21) $\frac{5.33 \times 10^{-6}}{9.62 \times 10^{-2}}$

22) $\frac{3.2 \times 10^{-3}}{5.02 \times 10^{0}}$

23) $(5.5 \times 10^{-5})^{2}$

24) $(9.6 \times 10^{3})^{-4}$

25) $(7.8 \times 10^{-2})^{5}$

26) $(5.4 \times 10^{6})^{-3}$

27) $(8.03 \times 10^{4})^{-4}$

28) $(6.88 \times 10^{-4})(4.23 \times 10^{1})$

29) $\frac{6.1 \times 10^{-6}}{5.1 \times 10^{-4}}$

30) $\frac{8.4 \times 10^{5}}{7 \times 10^{-2}}$

31) $(3.6 \times 10^{0})(6.1 \times 10^{-3})$

32) $(3.15 \times 10^{3})(8 \times 10^{-1})$

33) $(1.8 \times 10^{-5})^{-3}$

34) $\frac{9.58 \times 10^{-2}}{1.14 \times 10^{-3}}$

35) $\frac{9 \times 10^{4}}{7.83 \times 10^{-2}}$

36) $(8.3 \times 10^{1})^{5}$

37) $\frac{3.22 \times 10^{-3}}{7 \times 10^{-6}}$

38) $\frac{5 \times 10^{6}}{6.69 \times 10^{2}}$

39) $\frac{2.4 \times 10^{-6}}{6.5 \times 10^{0}}$

40) $(9 \times 10^{-2})^{-3}$

41) $\frac{6 \times 10^{3}}{5.8 \times 10^{-3}}$

42) $(2 \times 10^{4})(6 \times 10^{1})$

5.4

Polynomials - Introduction to Polynomials

Many applications in mathematics have to do with what are called polynomials. Polynomials are made up of terms. **Terms** are a product of numbers and/or variables. For example, $5x$, $2y^2$, -5, ab^3c, and x are all terms. Terms are connected to each other by addition or subtraction. Expressions are often named based on the number of terms in them. A **monomial** has one term, such as $3x^2$. A **binomial** has two terms, such as $a^2 - b^2$. A Trinomial has three terms, such as $\mathrm{ax}^2 + \mathrm{bx} + c$. The term **polynomial** means many terms. Monomials, binomials, trinomials, and expressions with more terms all fall under the umbrella of "polynomials". If we know what the variable in a polynomial represents we can replace the varaible with the number and evaluate the polynomial as shown in the following example.

Example 35.

$$\begin{array}{rl}
2x^2 - 4x + 6 \text{ when } x = -4 & \text{Replace varaible } x \text{ with } -4\\
2(-4)^2 - 4(-4) + 6 & \text{Exponents first}\\
2(16) - 4(-4) + 6 & \text{Multiplication (we can do all terms at once)}\\
32 + 16 + 6 & \text{Add}\\
54 & \text{Our Solution}
\end{array}$$

It is important to be careful with negative variables and exponents. Remember the exponent only effects the number it is physically attached to. This means $-3^2 = -9$ because the exponent is only attached to the 3. Also, $(-3)^2 = 9$ because the exponent is attached to the parenthesis and effects everything inside. When we replace a varible with parenthesis like in Example 1, the substituted value is in parenthesis. So the $(-4)^2 = 16$ in the example. However, consider the next example.

Example 36.

$$\begin{array}{rl}
-x^2 + 2x + 6 \text{ when } x = 3 & \text{Replace variable } x \text{ with } 3\\
-(3)^2 + 2(3) + 6 & \text{Exponent only on the 3, not negative}\\
-9 + 2(3) + 6 & \text{Multiply}
\end{array}$$

$$
\begin{array}{rl}
-9+6+6 & \text{Add} \\
3 & \text{Our Solution}
\end{array}
$$

Generally when working with polynomials we do not know the value of the variable, so we will try and simplify instead. The simplest operation with polynomials is addition. When adding polynomials we are mearly combining like terms. Consider the following example

Example 37.

$$
\begin{array}{rl}
(4x^3-2x+8)+(3x^3-9x^2-11) & \text{Combine like terms } 4x^3+3x^3 \text{ and } 8-11 \\
7x^3-9x^2-2x-3 & \text{Our Solution}
\end{array}
$$

Generally final answers for polynomials are written so the exponent on the variable counts down. Example 3 demonstrates this with the exponent counting down 3, 2, 1, 0 (recall $x^0=1$). Subtracting polynomials is almost as fast. One extra step comes from the minus in front of the parenthesis. When we have a negative in front of parenthesis we distribute it through, changing the signs of everything inside. The same is done for the subtraction sign.

Example 38.

$$
\begin{array}{rl}
(5x^2-2x+7)-(3x^2+6x-4) & \text{Distribute negative through second part} \\
5x^2-2x+7-3x^2-6x+4 & \text{Combine like terms } 5x^2-3x^3, -2x-6x, \text{ and } 7+4 \\
2x^2-8x+11 & \text{Our Solution}
\end{array}
$$

Addition and subtraction can also be combined into the same problem as shown in this final example.

Example 39.

$$
\begin{array}{rl}
(2x^2-4x+3)+(5x^2-6x+1)-(x^2-9x+8) & \text{Distribute negative through} \\
2x^2-4x+3+5x^2-6x+1-x^2+9x-8 & \text{Combine like terms} \\
6x^2-x-4 & \text{Our Solution}
\end{array}
$$

Practice - Add and Subtract Polynomials

Simplify each expression.

1) $f(a) = -a^3 - a^2 + 6a - 21$ at $a = -4$

2) $f(n) = n^2 + 3n - 11$ at $n = -6$

3) $f(n) = n^3 - 7n^2 + 15n - 20$ at $n = 2$

4) $f(n) = n^3 - 9n^2 + 23n - 21$ at $n = 5$

5) $f(n) = -5n^4 - 11n^3 - 9n^2 - n - 5$ at $n = -1$

6) $f(x) = x^4 - 5x^3 - x + 13$ at $x = 5$

7) $f(x) = x^2 + 9x + 23$ at $x = -3$

8) $f(x) = -6x^3 + 41x^2 - 32x + 11$ at $x = 6$

9) $f(x) = x^4 - 6x^3 + x^2 - 24$ at $x = 6$

10) $f(m) = m^4 + 8m^3 + 14m^2 + 13m + 5$ at $m = -6$

11) $(5p - 5p^4) - (8p - 8p^4)$

12) $(7m^2 + 5m^3) - (6m^3 - 5m^2)$

13) $(3n^2 + n^3) - (2n^3 - 7n^2)$

14) $(x^2 + 5x^3) + (7x^2 + 3x^3)$

15) $(8n + n^4) - (3n - 4n^4)$

16) $(3v^4 + 1) + (5 - v^4)$

17) $(1 + 5p^3) - (1 - 8p^3)$

18) $(6x^3 + 5x) - (8x + 6x^3)$

19) $(5n^4 + 6n^3) + (8 - 3n^3 - 5n^4)$

20) $(8x^2 + 1) - (6 - x^2 - x^4)$

21) $(3+b^4)+(7+2b+b^4)$

22) $(1+6r^2)+(6r^2-2-3r^4)$

23) $(8x^3+1)-(5x^4-6x^3+2)$

24) $(4n^4+6)-(4n-1-n^4)$

25) $(2a+2a^4)-(3a^2-5a^4+4a)$

26) $(6v+8v^3)+(3+4v^3-3v)$

27) $(4p^2-3-2p)-(3p^2-6p+3)$

28) $(7+4m+8m^4)-(5m^4+1+6m)$

29) $(4b^3+7b^2-3)+(8+5b^2+b^3)$

30) $(7n+1-8n^4)-(3n+7n^4+7)$

31) $(3+2n^2+4n^4)+(n^3-7n^2-4n^4)$

32) $(7x^2+2x^4+7x^3)+(6x^3-8x^4-7x^2)$

33) $(n-5n^4+7)+(n^2-7n^4-n)$

34) $(8x^2+2x^4+7x^3)+(7x^4-7x^3+2x^2)$

35) $(8r^4-5r^3+5r^2)+(2r^2+2r^3-7r^4+1)$

36) $(4x^3+x-7x^2)+(x^2-8+2x+6x^3)$

37) $(2n^2+7n^4-2)+(2+2n^3+4n^2+2n^4)$

38) $(7b^3-4b+4b^4)-(8b^3-4b^2+2b^4-8b)$

39) $(8-b+7b^3)-(3b^4+7b-8+7b^2)+(3-3b+6b^3)$

40) $(1-3n^4-8n^3)+(7n^4+2-6n^2+3n^3)+(4n^3+8n^4+7)$

41) $(8x^4+2x^3+2x)+(2x+2-2x^3-x^4)-(x^3+5x^4+8x)$

42) $(6x-5x^4-4x^2)-(2x-7x^2-4x^4-8)-(8-6x^2-4x^4)$

5.5

Polynomials - Multiplying

Multiplying polynomials can take several different forms based on what we are multiplying. We will first look at multiplying monomials, then monomials by polynomials and finish with polynomials by polynomials.

Multiplying monomials is done by multiplying the numbers or coefficients and then adding the exponents on like factors. This is shown in the next example.

Example 40.

$$\begin{array}{rl} (4x^3y^4z)(2x^2y^6z^3) & \text{Multiply numbers and add exponents for } x, y, \text{ and } z \\ 8x^5y^{10}z^4 & \text{Our Solution} \end{array}$$

In the previous example it is important to remember that the z has an exponent of 1 when no exponent is written. Thus for our answer the z has an exponent of $1+3=4$. Be very careful with exponents in polynomials. If we are adding or subtracting the exponnets will stay the same, but when we multiply (or divide) the exponents will be changing.

Next we consider multiplying a monomial by a polynomial. We have seen this operation before with distributing through parenthesis. Here we will see the exact same process.

Example 41.

$$\begin{array}{rl} 4x^3(5x^2-2x+5) & \text{Distribute the } 4x^3\text{, multiplying numbers, adding exponents} \\ 20x^5-2x^4+5x^3 & \text{Our Solution} \end{array}$$

Following is another example with more variables. When distributing the exponents on a are added and the exponents on b are added.

Example 42.

$$\begin{array}{rl} 2a^3b(3\mathrm{ab}^2-4a) & \text{Distribute, multiplying numbers and adding exponents} \\ 6a^4b^3-8a^4b & \text{Our Solution} \end{array}$$

There are several different methods for multiplying polynomials. All of which work, often students prefer the method they are first taught. Here three methods will be discussed. All three methods will be used to solve the same two multiplication problems.

Multiply by Distributing

Just as we distribute a monomial through parenthesis we can distribute an entire polynomial. As we do this we take each term of the second polynomial and put it in front of the first polynomial.

Example 43.

$$\begin{array}{rl} (4x+7y)(3x-2y) & \text{Distribute } (4x+7y) \text{ through parenthesis} \\ 3x\boldsymbol{(4x+7y)}-2y\boldsymbol{(4x+7y)} & \text{Distribute the } 3x \text{ and } -2y \\ 12x^2+21xy-8xy-14y^2 & \text{Combine like terms } 21xy-8xy \\ 12x^2+13xy-14y^2 & \text{Our Solution} \end{array}$$

Example 4 illustrates an important point, the negative/subtraction sign stays with the $2y$. Which means on the second step the negative is also distributed through the last set of parenthesis.

Multiplying by distributing can easily be extended to problems with more terms. First distribute the front parenthesis onto each term, then distribute again!

Example 44.

$$\begin{array}{rl} (2x-5)(4x^2-7x+3) & \text{Distribute } (2x-5) \text{ through parenthesis} \\ 4x^2\boldsymbol{(2x-5)}-7x\boldsymbol{(2x-5)}+3\boldsymbol{(2x-5)} & \text{Distribute again through each parenthesis} \\ 8x^3-20x^2-14x^2+35x+6x-15 & \text{Combine like terms} \\ 8x^3-34x^2+41x-15 & \text{Our Solution} \end{array}$$

This process of multiplying by distributing can easily be reversed to do an important proceedure known as factoring. Factoring will be addressed in a future lesson.

Multiply by FOIL

Another form of multiplying is known as FOIL. Using the FOIL method we multiply each term in the first binomial by each term in the second binomial. The letters of FOIL help us remember every combination. F stands for First, we multiply the first term of each binomial. O stand for Outside, we multiply the outside two terms. I stands for Inside, we multiply the inside two terms. L stands for Last, we multiply the last term of each binomial. This is shown in the next example:

Example 45.

$$\begin{array}{rl}
(4x+7y)(3x-2y) & \text{Use FOIL to multiply} \\
(4x)(3x)=12x^2 & F-\text{First terms }(4x)(3x) \\
(4x)(-2y)=-8xy & O-\text{Outside terms }(4x)(-2y) \\
(7y)(3x)=21xy & I-\text{Inside terms }(7y)(3x) \\
(7y)(-2y)=-14y^2 & L-\text{Last terms }(7y)(-2y) \\
12x^2-8xy+21xy-14y^2 & \text{Combine like terms }-8xy+21xy \\
12x^2+13xy-14y^2 & \text{Our Solution}
\end{array}$$

Some student like to think of the FOIL method as distributing the first term $4x$ through the $(3x-2y)$ and distributing the second term $7y$ through the $(3x-2y)$. Thinking about FOIL in this way makes it possible to extend this method to problems with more terms.

Example 46.

$$\begin{array}{rl}
(2x-5)(4x^2-7x+3) & \text{Distribute }2x\text{ and }-5 \\
(2x)(4x^2)+(2x)(-7x)+(2x)(3)-5(4x^2)-5(-7x)-5(3) & \text{Multiply out each term} \\
8x^3-14x^2+6x-20x^2+35x-15 & \text{Combine like terms} \\
8x^3-34x^2+41x-15 & \text{Our Solution}
\end{array}$$

The second step of the FOIL method is often not written, for example, consider example 6, a student will often go from the problem $(4x+7y)(3x-2y)$ and do the multiplication metally to come up with $12x^2-8xy+21xy-14y^2$ and then combine like terms to come up with the final solution.

Multiplying in rows

A third method for multiplying polynomials looks very similar to multiplying numbers. consider the problem:

$$\begin{array}{rl}
35 & \\
\underline{\times 27} & \\
245 & \text{Multiply 7 by 5 then 3} \\
\underline{700} & \text{Use 0 for placeholder, multiply 2 by 5 then 3} \\
945 & \text{Add to get our solution}
\end{array}$$

The same process can be done with polynomials. Multiply each term on the bottom with each term on the top.

Example 47.

$$\begin{array}{rl}
(4x+7y)(3x-2y) & \text{Rewrite as vertical problem} \\
4x+7y & \\
\underline{\times\, 3x-2y} & \\
-8xy-14y^2 & \text{Multiply } -2y \text{ by } 7y \text{ then } 4x \\
\underline{12x^2+21xy\qquad\quad} & \text{Multiply } 3x \text{ by } 7y \text{ then } 4x\text{. Line up like terms} \\
12x^2+13xy-14y^2 & \text{Add like terms to get our solution}
\end{array}$$

This same process is easily expanded to a problem with more terms.

Example 48.

$$\begin{array}{rl}
(2x-5)(4x^2-7x+3) & \text{Rewrite as vertical problem} \\
4x^3-7x+3 & \text{Put polynomial with most terms on top} \\
\underline{\times\, 2x-5} & \\
-20x^2+35x-15 & \text{Multiply } -5 \text{ by each term} \\
\underline{8x^3-14x^2+6x\qquad\quad} & \text{Multiply } 2x \text{ by each term. Line up like terms} \\
8x^3-34x^2+41x-15 & \text{Add like terms to get our solution}
\end{array}$$

This method of multiplying in rows also works with multiplying a monomial by a polynomial!

Any of the three described methods work to multiply polynomials. It is suggested that you are very comfortable with at least one of these methods as you work through the practice problems. All three methods are shown side by side in Example 10

Example 49.

$$(2x-y)(4x-5y)$$

Distribute	**FOIL**	**Rows**
$4x(2x-y)-5y(2x-y)$	$2x(4x)+2x(-5y)-y(4x)-y(-5y)$	$2x-y$
$8x^2-4xy-10xy-5y^2$	$8x^2-10xy-4xy+5y^2$	$\underline{\times\, 4x-5y}$
$8x^2-14xy-5y^2$	$8x^2-14xy+5y^2$	$-10xy+5y^2$
		$\underline{8x^2-4xy\qquad\quad}$
		$8x^2-14xy+5y^2$

Practice - Multiply Polynomials

Find each product.

1) $6(p-7)$

2) $4k(8k+4)$

3) $2(6x+3)$

4) $3n^2(6n+7)$

5) $5m^4(4m+4)$

6) $3(4r-7)$

7) $(4n+6)(8n+8)$

8) $(2x+1)(x-4)$

9) $(8b+3)(7b-5)$

10) $(r+8)(4r+8)$

11) $(4x+5)(2x+3)$

12) $(7n-6)(n+7)$

13) $(3v-4)(5v-2)$

14) $(6a+4)(a-8)$

15) $(6x-7)(4x+1)$

16) $(5x-6)(4x-1)$

17) $(5x+y)(6x-4y)$

18) $(2u+3v)(8u-7v)$

19) $(x+3y)(3x+4y)$

20) $(8u+6v)(5u-8v)$

21) $(7x+5y)(8x+3y)$

22) $(5a+8b)(a-3b)$

23) $(r-7)(6r^2-r+5)$

24) $(4x+8)(4x^2+3x+5)$

25) $(6n-4)(2n^2-2n+5)$

26) $(2b-3)(4b^2+4b+4)$

27) $(6x+3y)(6x^2-7xy+4y^2)$

28) $(3m-2n)(7m^2+6mn+4n^2)$

29) $(8n^2+4n+6)(6n^2-5n+6)$

30) $(2a^2+6a+3)(7a^2-6a+1)$

31) $(5k^2+3k+3)(3k^2+3k+6)$

32) $(7u^2+8\text{uv}-6v^2)(6u^2+4uv+3v^2)$

5.6

Polynomials - Special Products

There are a few shortcuts that we can take when multiplying polynomials. If we can recognize them the shortcuts can help us arrive at the solution much quicker. These shortcuts will also be useful to us as our study of algebra continues.

The first shortcut is often called a **sum and a difference**. A sum and a difference is easily recognized as the numbers and variables are exactly the same, but the sign in the middle is different (one sum, one difference). To illustrate the shortcut consider the following example, multiplied by the distributing method.

Example 50.

$$\begin{array}{rl} (a+b)(a-b) & \text{Distribute } (a+b) \\ a(a+b)-b(a+b) & \text{Distribute } a \text{ and } -b \\ a^2+ab-ab-b^2 & \text{Combine like terms } ab-ab \\ a^2-b^2 & \text{Our Solution} \end{array}$$

The important part of this example is the middle terms subtracted to zero. Rather than going through all this work, when we have a sum and a difference we will jump right to our solution by squaring the first term and squaring the last term, putting a subtraction between them. This is illustrated in the following example

Example 51.

$$\begin{array}{rl} (x-5)(x+5) & \text{Recognize sum and difference} \\ x^2-25 & \text{Square both, put subtraction between. Our Solution} \end{array}$$

This is much quicker than going through the work of multiplying and combining like terms. Often students ask if they can just multiply out using another method and not learn the shortcut. These shortcuts are going to be very useful when we get to factoring polynomials, or reversing the multiplication process. For this reason it is very important to be able to recognize these shortcuts. More examples are shown here.

Example 52.

$$
\begin{array}{rl}
(3x+7)(3x-7) & \text{Recognize sum and difference} \\
9x^2-49 & \text{Square both, put subtraction between. Our Solution}
\end{array}
$$

Example 53.

$$
\begin{array}{rl}
(2x-6y)(2x+6y) & \text{Recognize sum and difference} \\
4x^2-36y^2 & \text{Square both, put subtraction between. Our Solution}
\end{array}
$$

It is interesting to note that while we can multiply and get an answer like a^2-b^2 (with subtraction), it is impossible to multiply real numbers and end up with a product such as a^2+b^2 (with addition).

Another shortcut used to multiply is known as a **perfect square**. These are easy to recognize as we will have a binomial with a 2 in the exponent. The following example illustrates multiplying a perfect square

Example 54.

$$
\begin{array}{rl}
(a+b)^2 & \text{Squared is same as multiplying by itself} \\
(a+b)(a+b) & \text{Distribute } (a+b) \\
a(a+b)+b(a+b) & \text{Distribute again through final parenthesis} \\
a^2+ab+ab+b^2 & \text{Combine like terms } ab+ab \\
a^2+2\text{ab}+b^2 & \text{Our Solution}
\end{array}
$$

This problem also helps us find our shortcut for multiplying. The first term in the answer is the square of the first term in the problem. The middle term is 2 times the first term times the second term. The last term is the square of the last term. This can be shortened to square the first, twice the product, square the last. If we can remember this shortcut we can square any binomial. This is illustrated in the following example

Example 55.

$$
\begin{array}{rl}
(x-5)^2 & \text{Recognize perfect square} \\
x^2 & \text{Square the first} \\
2(x)(-5)=-10x & \text{Twice the product} \\
(-5)^2=25 & \text{Square the last} \\
x^2-10x+25 & \text{Our Solution}
\end{array}
$$

Be very careful when we are squaring a binomial to **NOT** distribute the square through the parenthesis. A common error is to do the following: $(x-5)^2 = x^2 - 25$ (or $x^2 + 25$). Notice both of these are missing the middle term, $-10x$. This is why it is important to use the shortcut to help us find the correct solution. Another important observation is that the middle term in the solution always has the same sign as the middle term in the problem. This is illustrated in the next examples.

Example 56.

$$\begin{array}{rl} (2x+5)^2 & \text{Recognize perfect square} \\ (2x)^2 = 4x^2 & \text{Square the first} \\ 2(2x)(5) = 20x & \text{Twice the product} \\ 5^2 = 25 & \text{Square the last} \\ 4x^2 + 20x + 25 & \text{Our Solution} \end{array}$$

Example 57.

$$\begin{array}{rl} (3x-7y)^2 & \text{Recognize perfect square} \\ 9x^2 - 42\text{xy} + 49y^2 & \text{Square the first, twice the product, square the last. Our Solution} \end{array}$$

Example 58.

$$\begin{array}{rl} (5a+9b)^2 & \text{Recognize perfect square} \\ 25a^2 + 90\text{ab} + 81b^2 & \text{Square the first, twice the product, square the last. Our Solution} \end{array}$$

These two formulas will be important to commit to memory. The more familiar we are with the easier factoring, or multiplying in reverse, will be. The final example covers both types of problems (two perfect squares, on positive, on negative), be sure to notice the difference between the examples and how each formula is used

Example 59.

$$\begin{array}{ccc} (4x-7)(4x+7) & (4x+7)^2 & (4x-7)^2 \\ 16x^2 - 49 & 16x^2 + 56x + 49 & 16x^2 - 56x + 49 \end{array}$$

Practice - Multiply Special Products

Find each product.

1) $(x+8)(x-8)$

2) $(a-4)(a+4)$

3) $(1+3p)(1-3p)$

4) $(x-3)(x+3)$

5) $(1-7n)(1+7n)$

6) $(8m+5)(8m-5)$

7) $(5n-8)(5n+8)$

8) $(2r+3)(2r-3)$

9) $(4x+8)(4x-8)$

10) $(b-7)(b+7)$

11) $(4y-x)(4y+x)$

12) $(7a+7b)(7a-7b)$

13) $(4m-8n)(4m+8n)$

14) $(3y-3x)(3y+3x)$

15) $(6x-2y)(6x+2y)$

16) $(1+5n)^2$

17) $(a+5)^2$

18) $(v+4)^2$

19) $(x-8)^2$

20) $(1-6n)^2$

21) $(p+7)^2$

22) $(7k-7)^2$

23) $(7-5n)^2$

24) $(4x-5)^2$

25) $(5m-8)^2$

26) $(3a+3b)^2$

27) $(5x+7y)^2$

28) $(4m-n)^2$

29) $(2x+2y)^2$

30) $(8x+5y)^2$

31) $(5+2r)^2$

32) $(m-7)^2$

33) $(2+5x)^2$

34) $(8n+7)(8n-7)$

35) $(4v-7)\,(4v+7)$

36) $(b+4)(b-4)$

37) $(n-5)(n+5)$

38) $(7x+7)^2$

39) $(4k+2)^2$

40) $(3a-8)(3a+8)$

5.7

Polynomials - Dividing

Dividing polynomials is a process very similar to long division of whole numbers. But before we look at that, we will first want to be able to master dividing a polynomial by a monomial. The way we do this is very similar to distributing, but the operation we distribute is the division, dividing each term by the monomial and reducing the resulting expression. This is shown in the following examples

Example 60.

$$\frac{9x^5+6x^4-18x^3-24x^2}{3x^2} \quad \text{Divide each term in the numerator by } 3x^2$$

$$\frac{9x^5}{3x^2}+\frac{6x^4}{3x^2}-\frac{18x^3}{3x^2}-\frac{24x^2}{3x^2} \quad \text{Reduce each fraction, subtracting exponents}$$

$$3x^3+2x^2-6x-8 \quad \text{Our Solution}$$

Example 61.

$$\frac{8x^3+4x^2-2x+6}{4x^2} \quad \text{Divide each term in the numerator by } 4x^2$$

$$\frac{8x^3}{4x^2}+\frac{4x^2}{4x^2}-\frac{2x}{4x^2}+\frac{6}{4x^2} \quad \text{Reduce each fraction, subtracting exponents,}$$

Remember negative exponents are moved to denominator

$$2x+1-\frac{1}{2x}+\frac{3}{2x^2} \quad \text{Our Solution}$$

The previous example illustrates that sometimes will will have fractions in our solution, as long as they are reduced this will be correct for our solution. Also interesting in this problem is the second term $\frac{4x^2}{4x^2}$ divided out completely. Remember that this means the reduced answer is 1 not 0.

Long division is required when we divide by more than just a monomial. Long division with polynomials works very similar to long division with whole numbers. An example is given to review the (general) steps that are used with whole numbers that we will also use with polynomials

Example 62.

$4\overline{\smash{)}631}$	Divide front numbers: $\frac{6}{4}=1\ldots$
$\mathbf{1}$	
$4\overline{\smash{)}631}$	Multiply this number by divisor: $1\cdot 4=4$
$\underline{-\mathbf{4}}$	Change the sign of this number (make it subtract) and combine
$\mathbf{23}$	Bring down next number
$1\mathbf{5}$	Repeat, divide front numbers: $\frac{23}{4}=5\ldots$
$4\overline{\smash{)}631}$	
$\underline{-4}$	
23	Multiply this number by divisor: $5\cdot 4=20$
$\underline{-\mathbf{20}}$	Change the sign of this number (make it subtract) and combine
$\mathbf{31}$	Bring down next number
$15\mathbf{7}$	Repeat, divide front numbers: $\frac{31}{4}=7\ldots$
$4\overline{\smash{)}631}$	
$\underline{-4}$	
23	
$\underline{-20}$	
31	Multiply this number by divisor: $7\cdot 4=28$
$\underline{-\mathbf{28}}$	Change the sign of this number (make it subtract) and combine
$\mathbf{3}$	We will write our remainder as a fraction, over the divisor, added to the end
$157\frac{3}{4}$	Our Solution

This same process will be used to multiply polynomials. The only difference is we will replace the word “number” with the word “term”

Dividing Polynomials

1. Divide front terms
2. Multiply this term by the divisor
3. Change the sign of the terms and combine

4. Bring down the next term

5. Repeate

Step number 3 tends to be the one that students skip, not changing the signs of the terms would be equivalent to adding instead of subtracting on long division with whole numbers. Be sure not to miss this step! This process is illustrated in the following two examples.

Example 63.

$$\begin{array}{rl}
\dfrac{3x^3-5x^2-32x+7}{x-4} & \text{Rewrite problem as long division}
\end{array}$$

$$\begin{array}{rl}
x-4\overline{)3x^3-5x^2-32x+7} & \text{Divide front terms: } \dfrac{3x^3}{x}=3x^2
\end{array}$$

$$\begin{array}{rl}
\boldsymbol{3x^2}\qquad\qquad\qquad & \\
x-4\overline{)3x^3-5x^2-32x+7} & \text{Multiply this term by divisor: } 3x^2(x-4)=3x^3-12x^2 \\
\underline{\boldsymbol{-3x^3+12x^2}\qquad\quad} & \text{Change the signs and combine} \\
\boldsymbol{7x^2-2x}\quad & \text{Bring down the next term}
\end{array}$$

$$\begin{array}{rl}
3x^2+\boldsymbol{7x}\qquad\qquad & \text{Repeate, divide front terms: } \dfrac{7x^2}{x}=7x \\
x-4\overline{)3x^3-5x^2-32x+7} & \\
\underline{-3x^3+12x^2\qquad\quad} & \\
7x^2-32x\quad & \text{Multiply this term by divisor: } 7x(x-4)=7x^2-28x \\
\underline{\boldsymbol{-7x^2+28x}\quad} & \text{Change the signs and combine} \\
\boldsymbol{-4x+7} & \text{Bring down the next term}
\end{array}$$

$$\begin{array}{rl}
3x^2+7x\boldsymbol{-4}\qquad\qquad & \text{Repeate, divide front terms: } \dfrac{-4x}{x}=-4 \\
x-4\overline{)3x^3-5x^2-32x+7} & \\
\underline{-3x^3+12x^2\qquad\quad} & \\
7x^2-32x\quad & \\
\underline{-7x^2+28x\quad} & \\
-4x+7 & \text{Multiply this term by divisor: } -4(x-4)=-4x+16 \\
\underline{\boldsymbol{+4x-16}} & \text{Change the signs and combine} \\
\boldsymbol{-9} & \text{Remainder put over divisor and subtracted (due to negative)}
\end{array}$$

$$3x^2+7x-4-\frac{9}{x-4} \quad \text{Our Solution}$$

Example 64.

$$\frac{6x^3-8x^2+10x+103}{2x+4} \quad \text{Rewrite problem as long division}$$

$$2x+4\overline{)6x^3-8x^2+10x+103} \quad \text{Divide front terms: } \frac{6x^3}{2x}=3x^2$$

$$\begin{array}{rl}
\mathbf{3x^2} & \\
2x+4\overline{)6x^3-8x^2+10x+103} & \text{Multiply term by divisor: } 3x^2(2x+4)=6x^3+12x^2 \\
\underline{\boldsymbol{-6x^3-12x^2}} & \text{Change the signs and combine} \\
\boldsymbol{-20x^2+10x} & \text{Bring down the next term}
\end{array}$$

$$\begin{array}{rl}
3x^2\,\boldsymbol{-10x} & \\
2x+4\overline{)6x^3-8x^2+10x+103} & \text{Repeate, divide front terms: } \frac{-20x^2}{2x}=-10x \\
\underline{-6x^3-12x^2} & \text{Multiply this term by divisor:} \\
-20x^2+10x & -10x(2x+4)=-20x^2-40x \\
\underline{\boldsymbol{+20x^2+40x}} & \text{Change the signs and combine} \\
\boldsymbol{50x+103} & \text{Bring down the next term}
\end{array}$$

$$\begin{array}{rl}
3x^2-10x\,\boldsymbol{+25} & \\
2x+4\overline{)6x^3-8x^2+10x+103} & \text{Repeate, divide front terms: } \frac{50x}{2x}=25 \\
\underline{-6x^3-12x^2} & \\
-20x^2+10x & \\
\underline{+20x^2+40x} & \\
50x+103 & \text{Multiply this term by divisor: } 25(2x+4)=50x+100 \\
\underline{\boldsymbol{-50x-100}} & \text{Change the signs and combine} \\
3 & \text{Remainder is put over divsor and added (due to positive)}
\end{array}$$

$$3x^2-10x+25+\frac{3}{2x+4} \quad \text{Our Solution}$$

In both of the previous example the dividends had the exponents on our variable counting down, no exponent skipped, third power, second power, first power, zero power (remember $x^0=1$ so there is no variable on zero power). This is very important in long division, the variables must count down and no exponent can be skipped. If they don't count down we must put them in order. If an exponent is skipped we will have to add a term to the problem, with zero for its coefficient. This is demonstrated in the following example.

Example 65.

$$\frac{2x^3+42-4x}{x+3} \qquad \text{Reorder dividend, need } x^2 \text{ term, add } 0x^2 \text{ for this}$$

$$x+3\overline{)2x^3+\mathbf{0x^2}-4x+42} \qquad \text{Divide front terms: } \frac{2x^3}{x}=2x^2$$

$$\begin{array}{rl}
\mathbf{2x^2} & \\
x+3\overline{)2x^3+0x^2-4x+42} & \text{Multiply this term by divisor: } 2x^2(x+3)=2x^3+6x^2 \\
\underline{\mathbf{-2x^3-6x^2}} & \text{Change the signs and combine} \\
\mathbf{-6x^2-4x} & \text{Bring down the next term}
\end{array}$$

$$\begin{array}{rl}
2x^2\mathbf{-6x} & \\
x+3\overline{)2x^3+0x^2-4x+42} & \text{Repeate, divide front terms: } \frac{-6x^2}{x}=-6x \\
\underline{-2x^3-6x^2} & \\
-6x^2-4x & \text{Multiply this term by divisor: } -6x(x+3)=-6x^2-18x \\
\underline{\mathbf{+6x^2+18x}} & \text{Change the signs and combine} \\
\mathbf{14x+42} & \text{Bring down the next term}
\end{array}$$

$$\begin{array}{rl}
2x^2-6x\mathbf{+14} & \\
x+3\overline{)2x^3+0x^2-4x+42} & \text{Repeate, divide front terms: } \frac{14x}{x}=14 \\
\underline{-2x^3-6x^2} & \\
-6x^2-4x & \\
\underline{+6x^2+18x} & \\
14x+42 & \text{Multiply this term by divisor: } 14(x+3)=14x+42 \\
\underline{\mathbf{-14x-42}} & \text{Change the signs and combine} \\
0 & \text{No remainder}
\end{array}$$

$$2x^2-6x+14 \qquad \text{Our Solution}$$

It is important to take a moment to check each problem to verify that the exponents count down and no exponent is skipped. If so we will have to adjust the problem. Also, this final example illustrates, just as in regular long division, sometimes we have no remainder in a problem.

Practice - Dividing Polynomials

Divide.

1) $\frac{20x^2+x^3+2x^2}{4x^3}$

2) $\frac{5x^4+45x^3+4x^2}{9x}$

3) $\frac{20n^4+n^3+40n^2}{10n}$

4) $\frac{3k^3+4k^2+2k}{8k}$

5) $\frac{12x^4+24x^3+3x^2}{6x}$

6) $\frac{5p^4+16p^3+16p^2}{4p}$

7) $\frac{10n^4+50n^3+2n^2}{10n^2}$

8) $\frac{3m^4+18m^3+27m^2}{9m^2}$

9) $\frac{x^2-2x-71}{x+8}$

10) $\frac{r^2-3r-53}{r-9}$

11) $\frac{n^2+13n+32}{n+5}$

12) $\frac{b^2-10b+16}{b-7}$

13) $\frac{v^2-2v-89}{v-10}$

14) $\frac{x^2+4x-26}{x+7}$

15) $\frac{a^2-4a-38}{a-8}$

16) $\frac{x^2-10x+22}{x-4}$

17) $\frac{45p^2+56p+19}{9p+4}$

18) $\frac{48k^2-70k+16}{6k-2}$

19) $\frac{10x^2-32x+9}{10x-2}$

20) $\frac{n^2+7n+15}{n+4}$

21) $\frac{4r^2-r-1}{4r+3}$

22) $\frac{3m^2+9m-9}{3m-3}$

23) $\frac{n^2-4}{n-2}$

24) $\frac{2x^2-5x-8}{2x+3}$

25) $\frac{27b^2+87b+35}{3b+8}$

26) $\frac{3v^2-32}{3v-9}$

27) $\frac{4x^2-33x+28}{4x-5}$

28) $\frac{4n^2-23n-38}{4n+5}$

29) $\frac{a^3+15a^2+49a-55}{a+7}$

30) $\frac{8k^3-66k^2+12k+37}{k-8}$

31) $\frac{x^3-26x-41}{x+4}$

32) $\frac{x^3-16x^2+71x-56}{x-8}$

33) $\frac{3n^3+9n^2-64n-68}{n+6}$

34) $\frac{k^3-4k^2-6k+4}{k-1}$

35) $\frac{x^3-46x+22}{x+7}$

36) $\frac{2n^3+21n^2+25n}{2n+3}$

37) $\frac{9p^3+45p^2+27p-5}{9p+9}$

38) $\frac{8m^3-57m^2+42}{8m+7}$

39) $\frac{r^3-r^2-16r+8}{r-4}$

40) $\frac{2x^3+12x^2+4x-37}{2x+6}$

41) $\frac{12n^3+12n^2-15n-4}{2n+3}$

42) $\frac{24b^3-38b^2+29b-60}{4b-7}$

43) $\frac{4v^3-21v^2+6v+19}{4v+3}$

Answers to Exponent Properties

1) 4^9
2) 4^6
3) 2^4
4) 3^6
5) $12m^2n$
6) $12x^3$
7) $8m^6n^3$
8) x^3y^6
9) 3^{12}
10) 4^{12}
11) 4^8
12) 3^6
13) $4u^6v^4$
14) x^3y^3
15) $16a^{16}$
16) $16x^4y^4$
17) $\frac{1}{4^2}$
18) $\frac{1}{3^2}$
19) 3
20) 3^3
21) m^2
22) $\frac{xy^3}{4}$
23) $\frac{4x^2}{3y}$
24) $\frac{y^2}{4}$
25) $4x^{10}y^{14}$
26) $8u^{18}v^6$
27) $2x^{17}y^{16}$
28) $\frac{3}{u^3v^3}$
29) $\frac{1}{6x^2y^5}$
30) $\frac{4}{3a^3}$
31) 64
32) $\frac{2}{a^6}$
33) $\frac{1}{512x^{24}y^{45}}$
34) $\frac{y^5x^2}{2}$
35) $\frac{1}{m^{12}n^{12}}$
36) $\frac{n^{10}}{2m}$
37) $\frac{2x^2}{y^2}$
38) $\frac{2}{y^2}$
39) $2q^7r^8p$
40) $\frac{4x^2}{y^2z^7}$
41) $\frac{y^{16}}{x^4z^4}$
42) $\frac{256r^8}{q^4}$
43) $\frac{4xy^4}{z^3}$

Answers to Negative Exponents

1) $32x^8y^{10}$
2) $\frac{32b^{13}}{a^2}$
3) $\frac{2a^{15}}{b^{11}}$
4) $2x^3y^2$
5) $16x^4y^8$
6) 1
7) $y^{16}x^5$
8) $\frac{32}{m^5n^{15}}$
9) $\frac{2}{9y}$
10) $\frac{y^5}{2x^7}$
11) $\frac{1}{y^2x^3}$
12) $\frac{y^8x^5}{4}$
13) $\frac{u}{4v^6}$
14) $\frac{x^7y^2}{2}$
15) $\frac{u^2}{12v^5}$
16) $\frac{y}{2x^4}$
17) $\frac{2}{y^2}$
18) $\frac{a^{16}}{2b}$
19) $16a^{12}b^{12}$
20) $\frac{y^8x^4}{4}$
21) $\frac{1}{2n^3}$
22) $2x^{16}y^2$
23) $16n^6m^4$
24) $\frac{2x}{y^3}$
25) $\frac{1}{x^{15}y}$
26) $4y^4$
27) $\frac{u}{2v}$
28) $4y^5$
29) 8
30) $\frac{1}{2u^3v^5}$
31) $2y^5x^4$
32) $\frac{a^3}{2b^3}$
33) $\frac{1}{x^2y^{11}z}$
34) $\frac{a^2}{8c^{10}b^{12}}$

35) $\frac{1}{h^3kj^6}$

36) $\frac{x^{30}z^6}{16y^4}$

37) $\frac{2b^{14}}{a^{12}c^7}$

38) $\frac{m^{14}q^8}{4p^4}$

39) $\frac{x^2}{y^4z^4}$

40) $\frac{mn^7}{p^5}$

Answers to Operations with Scientific Notation

1) 8.85×10^2

2) 7.44×10^{-4}

3) 8.1×10^{-2}

4) 1.09×10^0

5) 3.9×10^{-2}

6) 1.5×10^4

7) 870000

8) 256

9) 0.0009

10) 50000

11) 2

12) 0.00006

13) 1.4×10^{-3}

14) 1.76×10^{-10}

15) 1.662×10^{-6}

16) 5.018×10^6

17) 1.56×10^{-3}

18) 4.353×10^8

19) 1.815×10^4

20) 9.836×10^{-1}

21) 5.541×10^{-5}

22) 6.375×10^{-4}

23) 3.025×10^{-9}

24) 1.177×10^{-16}

25) 2.887×10^{-6}

26) 6.351×10^{-21}

27) 2.405×10^{-20}

28) 2.91×10^{-2}

29) 1.196×10^{-2}

30) 1.2×10^7

31) 2.196×10^{-2}

32) 2.52×10^3

33) 1.715×10^{14}

34) 8.404×10^1

35) 1.149×10^6

36) 3.939×10^9

37) 4.6×10^2

38) 7.474×10^3

39) 3.692×10^{-7}

40) 1.372×10^3

41) 1.034×10^6

42) 1.2×10^6

Answers to Add and Subtract Polynomials

1) 3

2) 7

3) -10

4) -6

5) -7

6) 8

7) 5

8) -1

9) 12

10) -1

11) $3p^4 - 3p$

12) $-m^3 + 12m^2$

13) $-n^3 + 10n^2$

14) $8x^3 + 8x^2$

15) $5n^4 + 5n$

16) $2v^4 + 6$

17) $13p^3$

18) $-3x$

19) $3n^3 + 8$

20) $x^4 + 9x^2 - 5$

21) $2b^4 + 2b + 10$

22) $-3r^4 + 12r^2 - 1$

23) $-5x^4 + 14x^3 - 1$

24) $5n^4 - 4n + 7$

25) $7a^4 - 3a^2 - 2a$

26) $12v^3 + 3v + 3$

27) $p^2 + 4p - 6$

28) $3m^4 - 2m + 6$

29) $5b^3 + 12b^2 + 5$

30) $-15n^4 + 4n - 6$

31) $n^3 - 5n^2 + 3$

32) $-6x^4 + 13x^3$

33) $-12n^4 + n^2 + 7$

34) $9x^2 + 10x^2$

35) $r^4 - 3r^3 + 7r^2 + 1$

36) $10x^3 - 6x^2 + 3x - 8$

37) $9n^4 + 2n^3 + 6n^2$

38) $2b^4 - b^3 + 4b^2 + 4b$

39) $-3b^4 + 13b^3 - 7b^2 - 11b + 19$

40) $12n^4 - n^3 - 6n^2 + 10$

41) $2x^4 - x^3 - 4x + 2$

42) $3x^4 + 9x^2 + 4x$

Answers to Multiply Polynomials

1) $6p - 42$

2) $32k^2 + 16k$

3) $12x + 6$

4) $18n^3 + 21n^2$

5) $20m^5 + 20m^4$

6) $12r - 21$

7) $32n^2 + 80n + 48$

8) $2x^2 - 7x - 4$

9) $56b^2 - 19b - 15$

10) $4r^2 + 40r + 64$

11) $8x^2 + 22x + 15$

12) $7n^2 + 43n - 42$

13) $15v^2 - 26v + 8$

14) $6a^2 - 44a - 32$

15) $24x^2 - 22x - 7$

16) $20x^2 - 29x + 6$

17) $30x^2 - 14xy - 4y^2$

18) $16u^2 + 10uv - 21v^2$

19) $3x^2 + 13xy + 12y^2$

20) $40u^2 - 34uv - 48v^2$

21) $56x^2 + 61xy + 15y^2$

22) $5a^2 - 7ab - 24b^2$

23) $6r^3 - 43r^2 - 12r - 35$

24) $16x^3 + 44x^2 + 44x + 40$

25) $12n^3 - 20n^2 + 38n - 20$

26) $8b^3 - 4b^2 - 4b - 12$

27) $36x^3 - 24x^2y + 3xy^2 + 12y^3$

28) $21m^3 + 4m^2n - 8n^3$

29) $48n^4 - 16n^3 + 64n^2 - 6n + 36$

30) $14a^4 + 30a^3 - 13a^2 - 12a + 3$

31) $15k^4 + 24k^3 + 48k^2 + 27k + 18$

32) $42u^4 + 76u^3v + 17u^2v^2 - 18v^4$

Answers to Multiply Special Products

1) $x^2 - 64$

2) $a^2 - 16$

3) $1 - 9p^2$

4) $x^2 - 9$

5) $1 - 49n^2$

6) $64m^2 - 25$

7) $25n^2 - 64$

8) $4r^2 - 9$

9) $16x^2 - 64$

10) $b^2 - 49$

11) $16y^2 - x^2$

12) $49a^2 - 49b^2$

13) $16m^2 - 64n^2$

14) $9y^2 - 9x^2$

15) $36x^2 - 4y^2$

16) $1 + 10n + 25n^2$

17) $a^2 + 10a + 25$

18) $v^2 + 8v + 16$

19) $x^2 - 16x + 64$

20) $1 - 12n + 36n^2$

21) $p^2 + 14p + 49$

22) $49k^2 - 98k + 49$

23) $49 - 70n + 25n^2$

24) $16x^2 - 40x + 25$

25) $25m^2 - 80m + 64$

26) $9a^2 + 18ab + 9b^2$

27) $25x^2 + 70xy + 49y^2$

28) $16m^2 - 8mn + n^2$

29) $4x^2 + 8xy + 4y^2$

30) $64x^2 + 80xy + 25y^2$

31) $25 + 20r + 4r^2$

32) $m^2 - 14m + 49$

33) $4 + 20x + 25x^2$

34) $64n^2 - 49$

35) $16v^2 - 49$

36) $b^2 - 16$

37) $n^2 - 25$

38) $49x^2 + 98x + 49$

39) $16k^2+16k+4$

40) $9a^2-64$

Answers to Dividing Polynomials

1) $5x+\frac{1}{4}+\frac{1}{2x}$

2) $\frac{5x^3}{9}+5x^2+\frac{4x}{9}$

3) $2n^3+\frac{n^2}{10}+4n$

4) $\frac{3k^2}{8}+\frac{k}{2}+\frac{1}{4}$

5) $2x^3+4x^2+\frac{x}{2}$

6) $\frac{5p^3}{4}+4p^2+4p$

7) $n^2+5n+\frac{1}{5}$

8) $\frac{m^2}{3}+2m+3$

9) $x-10+\frac{9}{x+8}$

10) $r+6+\frac{1}{r-9}$

11) $n+8-\frac{8}{n+5}$

12) $b-3-\frac{5}{b-7}$

13) $v+8-\frac{9}{v-10}$

14) $x-3-\frac{5}{x+7}$

15) $a+4-\frac{6}{a-8}$

16) $x-6-\frac{2}{x-4}$

17) $5p+4+\frac{3}{9p+4}$

18) $8k-9-\frac{1}{3k-1}$

19) $x-3+\frac{3}{10x-2}$

20) $n+3+\frac{3}{n+4}$

21) $r-1+\frac{2}{4x+3}$

22) $m+4+\frac{1}{m-1}$

23) $n+2$

24) $x-4+\frac{4}{2x+3}$

25) $9b+5-\frac{5}{3b+8}$

26) $v+3-\frac{5}{3v-9}$

27) $x-7-\frac{7}{4x-5}$

28) $n-7-\frac{3}{4n+5}$

29) $a^2+8a-7-\frac{6}{a+7}$

30) $8k^2-2k-4+\frac{5}{k-8}$

31) $x^2-4x-10-\frac{1}{x+4}$

32) x^2-8x+7

33) $3n^2-9n-10-\frac{8}{n+6}$

34) $k^2-3k-9-\frac{5}{k-1}$

35) $x^2-7x+3+\frac{1}{x+7}$

36) $n^2+9n-1+\frac{3}{2n+3}$

37) $p^2+4p-1+\frac{4}{9p+9}$

38) $m^2-8m+7-\frac{7}{8m+7}$

39) $r^2+3r-4-\frac{8}{r-4}$

40) $x^2+3x-7+\frac{5}{2x+6}$

41) $6n^2-3n-3+\frac{5}{2n+3}$

42) $6b^2+b+9+\frac{3}{4b-7}$

43) $v^2-6v+6+\frac{1}{4v+3}$