Beginning and Intermediate Algebra

Chapter 3: Inequalities

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Chapter 3: Inequalities

3.1

Inequalities - Graphing and Solving

When we have an equation such as $x = 4$ we have a specific value for our variable. With inequalities we will give a range of values for our variable. To do this we will not use equals, but one of the following symbols:

$<$	Greater than
$\leqslant$	Greater than or equal to
$>$	Less than
$\geqslant$	Less than or equal to

If we have an expression such as $x < 4$, this means our variable can be any number smaller than 4 such as -2, 0, 3, 3.9 or even 3.999999999 as long as it is smaller than 4. If we have an an expression such as $x \geqslant -2$, this means our variable can be any number greater than or equal to -2, such as $5, 0, -1, -1.9999$, or even -2.

Because we don't have one set value for our variable, it is often useful to draw a picture of the solutions to the inequality on a number line. We will start from the value in the problem and bold the lower part of the number line if the variable is smaller than the number, and bold the upper part of the number line if the variable is larger. The value itself we will mark with brackets, either) or (for less than or greater than respectively, and] or [for less than or equal to or greater than or equal to respectively.

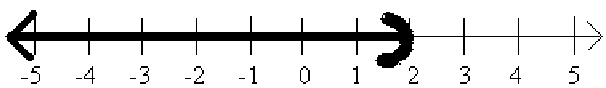
Once the graph is drawn we can quickly convert the graph into what is called interval notation. Interval notation gives two numbers, the first is the smallest value, the second is the largest value. If there is no largest value, we can use ∞ (infinity). If there is no smallest value, we can use $-\infty$ negative infinity. If we use either positive or negative infinity we will always use a curved bracket for that value.

Example 1.

Graph the inequality and give the interval notation

$x < 2$ Start at 2 and shade below

Use) for less than



Our Graph

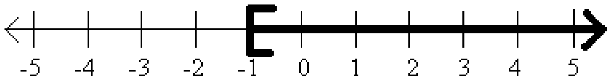
$(-\infty, 2)$ Interval Notation

Example 2.

Graph the inequality and give the interval notation

$$y \geqslant -1 \qquad \text{Start at } -1 \text{ and shade above}$$

Use [for greater than or equal



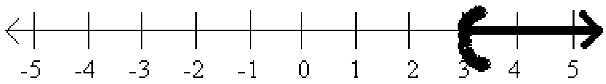
Our Graph

$[-1, \infty)$ Interval Notation

We can also take a graph and find the inequality for it.

Example 3.

Give the inequality for the graph:

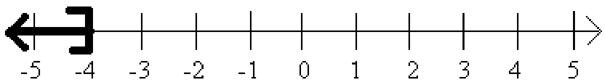


Graph starts at 3 and goes up or greater. Curved bracket means just greater than

$$x > 3 \qquad \text{Our Solution}$$

Example 4.

Give the inequality for the graph:



Graph starts at -4 and goes down or less. Square bracket means less than or equal to

$$x \leqslant -4 \qquad \text{Our Solution}$$

Generally when we are graphing and giving interval notation for an inequality we will have to first solve the inequality for our variable. Solving inequalities is very similar to solving equations with one exception. Consider the following inequality and what happens when various operations are done to it. Notice what happens to the inequality sign as we add, subtract, multiply and divide by both positive and negative numbers to keep the statment a true statement.

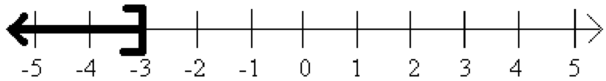
$$\begin{array}{rl}
5 > 1 & \text{Add 3 to both sides} \\
8 > 4 & \text{Subtract 2 from both sides} \\
6 > 2 & \text{Multiply both sides by 3} \\
12 > 6 & \text{Divide both sides by 2} \\
6 > 3 & \text{Add } -1 \text{ to both sides} \\
5 > 2 & \text{Subtract } -4 \text{ from both sides} \\
9 > 6 & \text{Multiply both sides by } -2 \\
-18 < -12 & \text{Divide both sides by } -6 \\
3 > 2 & \text{Symbol flipped when we multiply or divide by } a \text{ negative!}
\end{array}$$

As the above problem illustrates, we can add, subtract, multiply, or divide on both sides of the inequality. But if we multiply or divide by a negative number, the symbol will need to flip directions. We will keep that in mind as we solve inequalities.

Example 5.

Solve and give interval notation

$$\begin{array}{rl}
5-2x\geqslant 11 & \text{Subtract 5 from both sides} \\
\underline{-5 \qquad -5} & \\
-2x\geqslant 6 & \text{Divide both sides by } -2 \\
\overline{-2} \ \overline{-2} & \text{Divide by } a \text{ negative} - \text{flip symbol!} \\
x\leqslant -3 & \text{Graph, starting at } -3\text{, going down with] for less than or equal to}
\end{array}$$



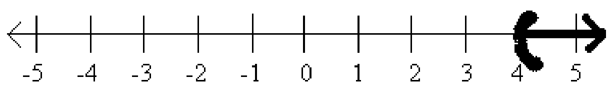
$$\begin{array}{rl}
(-\infty,-3] & \text{Interval Notation}
\end{array}$$

The inequality we solve can get as complex as the linear equations we solved. We will use all the same patterns to solve these inequalities as we did for solving equations. Just remember that any time we multiply or divide by a negative the symbol switches directions (multiplying or dividing by a positive does not change the symbol!)

Example 6.

Solve and give interval notation

$$\begin{array}{rl}
3(2x-4)+4x<4(3x-7)+8 & \text{Distribute} \\
6x-12+4x<12x-28+8 & \text{Combine like terms} \\
10x-12<12x-20 & \text{Move variable to one side} \\
\underline{-10x \qquad -10x} & \text{Subtract } 10x \text{ from both sides} \\
-12<2x-20 & \text{Add 20 to both sides} \\
\underline{+20 \qquad +20} & \\
8<2x & \text{Divide both sides by 2} \\
\overline{2} \ \overline{2} & \\
4<x & \text{Be careful with graph, } x \text{ is larger!}
\end{array}$$



$$\begin{array}{rl}
(4,\infty) & \text{Interval Notation}
\end{array}$$

It is important to be careful when the inequality is written backwards as in the previous example ($4<x$ rather than $x>4$). Often students draw their graphs the wrong way when this is the case. The inequality symbol opens to the variable, this means the variable is greater than 4. So we must shade above the 4.

Practice - Graphing and Solving Inequalities

Draw a graph for each inequality and give interval notation.

1) $n > -5$

2) $n > 4$

3) $-2 \geqslant k$

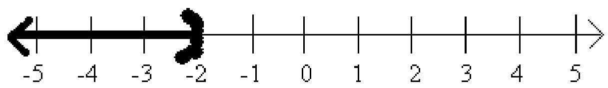
4) $1 \geqslant k$

5) $5 \geqslant x$

6) $-5 < x$

Write and inequality for each graph.

7)



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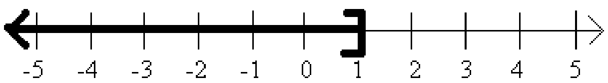

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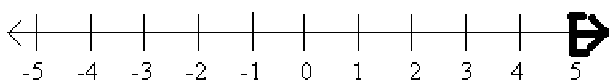

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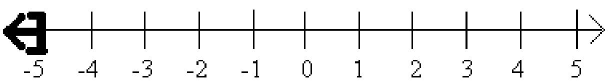

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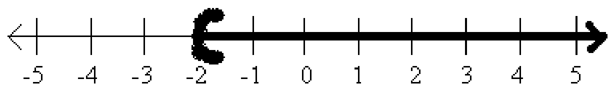

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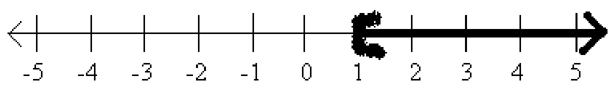


Solve each inequality, graph each solution, and give interval notation.

13) $\frac{x}{11} \geqslant 10$

14) $-2 \leqslant \frac{n}{13}$

15) $2 + r < 3$

16) $\frac{m}{5} \leqslant -\frac{6}{5}$

17) $8 + \frac{n}{3} \geqslant 6$

18) $11 > 8 + \frac{x}{2}$

19) $2 > \frac{a-2}{5}$

20) $\frac{v-9}{-4} \leqslant 2$

21) $-47 \geqslant 8 - 5x$

22) $\frac{6+x}{12} \leqslant -1$

23) $-2(3+k) < -44$

24) $-7n - 10 \geqslant 60$

25) $18 < -2(-8+p)$

26) $5 \geqslant \frac{x}{5} + 1$

27) $24 \geqslant -6(m-6)$

28) $-8(n-5) \geqslant 0$

29) $-r - 5(r-6) < -18$

30) $-60 \geqslant -4(-6x-3)$

31) $24 + 4b < 4(1+6b)$

32) $-8(2-2n) \geqslant -16 + n$

33) $-5v - 5 < -5(4v+1)$

34) $-36 + 6x > -8(x+2) + 4x$

35) $4 + 2(a+5) < -2(-a-4)$

36) $3(n+3) + 7(8-8n) < 5n + 5 + 2$

37) $-(k-2) > -k - 20$

38) $-(4-5p) + 3 \geqslant -2(8-5p)$

3.2

Inequalities - Compound Inequalities

Several inequalities can be combined together to form what are called compound inequalities. There are three types of compound inequalities which we will investigate in this lesson.

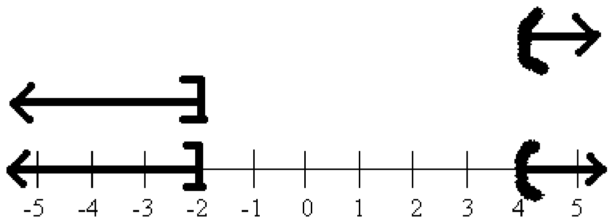
The first type of a compound inequality is an OR inequality. For this type of inequality we want a true statment from either one inequality OR the other inequality OR both. When we are graphing these type of inequalities we will graph each individule inequality above the number line, then move them both down together onto the actual number line for our graph that combines them together.

When we give interval notation for our solution, if there are two different parts to the graph we will put a $\cup$ (union) symbol between two sets of interval notation, one for each part.

Example 7.

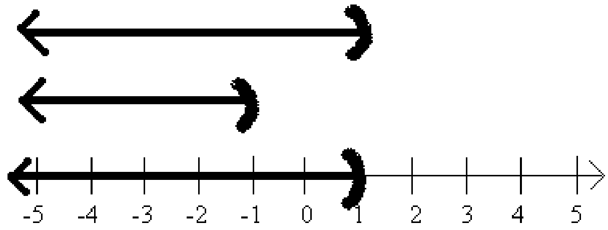
Solve each inequality, graph the solution, and give interval notation of solution

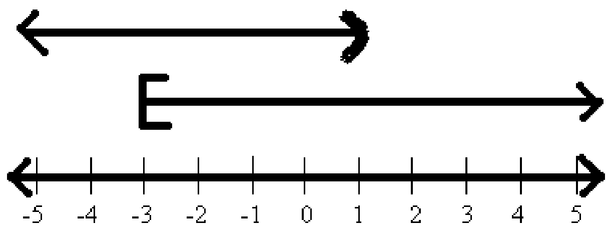
$$\begin{array}{rl}
2x-5>3 \text{ or } 4-x\geqslant 6 & \text{Solve each inequality} \\
\underline{+5+5} \quad \underline{-4 \qquad -4} & \text{Add or subtract first} \\
2x>8 \text{ or } -x\geqslant 2 & \text{Divide} \\
\overline{2} \;\; \overline{2} \qquad \overline{-1} \; \overline{-1} & \text{Dividing by negative flips sign} \\
x>4 \text{ or } x\leqslant -2 & \text{Graph the inequalities separatly above number line}
\end{array}$$



$$(-\infty,-2]\cup(4,\infty) \quad \text{Interval Notation}$$

There are several different results that could result from an OR statement. The graphs could be pointing different directions, as in the graph above, or pointing in the same direction as in the graph below on the left, or pointing opposite directions, but overlapping as in the graph below on the right. Notice how interval notation works for each of these cases.





As the graphs overlap, we take the largest graph for our solution.

Interval Notation: $(-\infty, 1)$

When the graphs are combined they cover the entire number line.

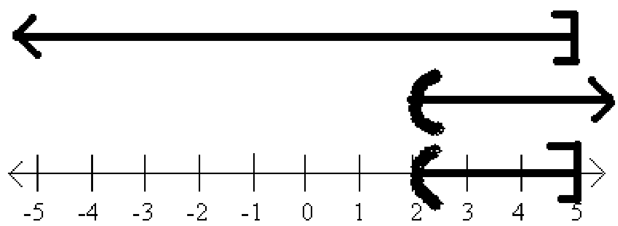
Internval Noation: $(-\infty, \infty)$ or $\mathbb{R}$

The second type of compound inequality is an AND inequality. AND inequalities require both statements to be true. If one is false, they both are false. When we graph these inequalities we can follow a similar process, first graph both inequalities above the number line, but this time only where they overlap will be drawn onto the number line for our final graph. When our solution is given in interval notation it will be expressed in a manner very similar to single inequalities (there is a symbol that can be used for AND, the intersection - $\cap$, but we will not use it here).

Example 8.

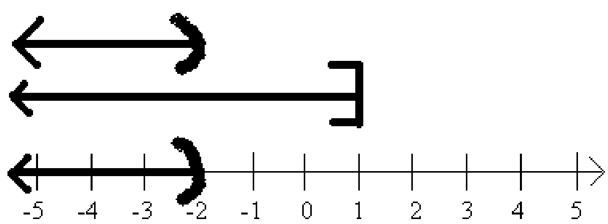
Solve each inequality, graph the solution, and express it interval notation.

$$\begin{array}{rcrl}
2x+8 \geqslant 5x-7 & \text{and} & 5x-3>3x+1 & \text{Move variables to one side} \\
\underline{-2x \quad -2x} & & \underline{-3x \quad -3x} & \\
8 \geqslant 3x-7 & \text{and} & 2x-3>1 & \text{Add 7 or 3 to both sides} \\
\underline{+7 \quad +7} & & \underline{+3+3} & \\
\frac{15}{3} \geqslant \frac{3x}{3} & \text{and} & \frac{2x}{2} > \frac{4}{2} & \text{Divide} \\
5 \geqslant x & \text{and} & x>2 & \text{Graph, } x \text{ is smaller (or equal) than 5, greater than 2}
\end{array}$$



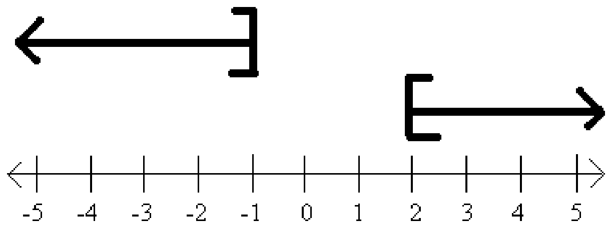
$$(2,5] \quad \text{Interval Notation}$$

Again, as we graph AND inequalities, only the overlaping parts of the individule graphs makes it to the final number line. As we graph AND inequalities there are also three different types of results we could get. The first is shown in the above example. The second is if the arrows both point the same way, this is shown below on the left. The third is if the arrows point opposite ways but don't overlap, this is shown below on the right. Notice how interval notation is expressed in each case.



In this graph, the overlap is only the smaller graph, so this is what makes it to the final number line.

Interval Notation: $(-\infty, -2)$



In this graph there is no overlap of the parts. Because their is no overlap, no values make it to the final number line.

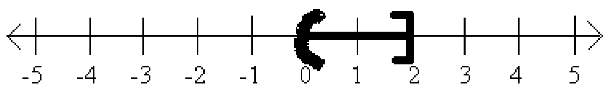
Interval Notation: No Solution or $\varnothing$

The third type of compound inequality is a special type of AND inequality. When our variable (or expression containing the variable) is between two numbers, we can write it as a single math sentence with three parts, such as $5 < x \leqslant 8$, to show x is between 5 and 8 (or equal to 8). When solving these type of inequalities, because there are three parts to work with, to stay balanced we will do the same thing to all three parts (rather than just both sides) to isolate the variable in the middle. The graph then is simply the values between the numbers with appropriate brackets on the ends.

Example 9.

Solve the inequality, graph the solution, and give interval notation.

$$\begin{array}{rl}
-6 \leqslant -4x + 2 < 2 & \text{Subtract 2 from all three parts} \\
\underline{-2 \qquad\quad -2 - 2} & \\
-8 \leqslant -4x < 0 & \text{Divide all three parts by } -4 \\
\overline{-4} \quad \overline{-4} \ \overline{-4} & \text{Dividing by } a \text{ negative flips the symbols} \\
2 \geqslant x > 0 & \text{Flip entire statment so values get larger left to right} \\
0 < x \leqslant 2 & \text{Graph } x \text{ between 0 and 2}
\end{array}$$



$(0, 2]$ Interval Notation

Practice - Compound Inequalities

Solve each compound inequality, graph its solution, and give interval notation.

1) $\frac{n}{3} \leqslant -3 \text{ or } -5n \leqslant -10$

2) $6m \geqslant -24 \text{ or } m - 7 < -12$

3) $x + 7 \geqslant 12 \text{ or } 9x < -45$

4) $10r > 0 \text{ or } r - 5 < -12$

5) $x - 6 < -13 \text{ or } 6x \leqslant -60$

6) $9 + n < 2 \text{ or } 5n > 40$

7) $\frac{v}{8} > -1 \text{ and } v - 2 < 1$

8) $-9x < 63 \text{ and } \frac{x}{4} < 1$

9) $-8 + b < -3 \text{ and } 4b < 20$

10) $-6n \leqslant 12 \text{ and } \frac{n}{3} \leqslant 2$

11) $a + 10 \geqslant 3 \text{ and } 8a < 48$

12) $-6 + v \geqslant 0 \text{ and } 2v > 4$

13) $3 \leqslant 9x \leqslant 7$

14) $0 \geqslant \frac{x}{9} \geqslant -1$

15) $11 < 8 + k \leqslant 12$

16) $-11 \leqslant n - 9 \leqslant -5$

17) $-3 < x - 1 < 1$

18) $1 \leqslant \frac{p}{8} \leqslant 0$

19) $-4 < 8 - 3m \leqslant 11$

20) $3 + 7r > 59 \text{ or } -6r - 3 > 33$

21) $-16 \leqslant 2n - 10 \leqslant -22$

22) $-6 - 8x \geqslant -6 \text{ or } 2 + 10x > 82$

23) $-5b + 10 \leqslant 30 \text{ and } 7b + 2 \leqslant -40$

24) $n + 10 \geqslant 15 \text{ or } 4n - 5 < -1$

25) $3x - 9 < 2x + 10 \text{ and } 5 + 7x \leqslant 10x - 10$

26) $-3n + 10 \leqslant -2n + 3 \leqslant 2 - 2n$

27) $-8 - 6v \leqslant 8 - 8v \text{ and } 7v + 9 \leqslant 6 + 10v$

28) $5 - 2a \geqslant 2a + 1 \text{ or } 10a - 10 \geqslant 9a + 9$

29) $1 + 5k \leqslant 7k - 3 \text{ or } k - 10 > 2k + 10$

30) $p - 8 \leqslant 2p + 7 < -2 - 7p$

31) $2x + 9 \geqslant 10x + 1 \text{ and } 3x - 2 < 7x + 2$

32) $-9m + 2 < -10 - 6m \text{ or } -m + 5 \geqslant 10 + 4m$

33) $4n + 8 < 3n - 6 \text{ or } 10n - 8 \geqslant 9 + 9n$

34) $8 - 10r \leqslant 8 + 4r \text{ or } -6 + 8r < 2 + 8r$

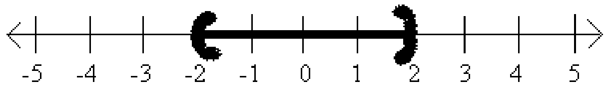
3.3

Inequalities - Absolute Value

When an inequality has an absolute value we will have to remove the absolute value in order to graph the solution or give interval notation. The way we remove the absolute value depends on the direction of the inequality symbol.

Consider $|x| < 2$.

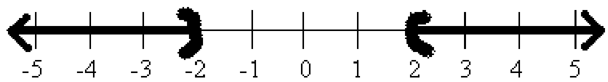
Absolute value is defined as distance from zero. Another way to read this inequality would be the distance from zero is less than 2. So on a number line we will shade all points that are less than 2 units away from zero.



This graph looks just like the graphs of the three part compound inequalities! When the absolute value is **less than** a number will will remove the absolute value by changing the problem to a three part inequality, with the negative value on the left and the positive value on the right. So $|x| < 2$ becomes $-2 < x < 2$, as the graph above illustrates.

Consider $|x| > 2$.

Absolute value is defined as distance from zero. Another way to read this inequality would be the distance from zero is greater than 2. So on the number line we shade all points that are more than 2 units away from zero.



This graph looks just like the graphs of the OR compound inequalities! When the absolute value is **greater than** a number we will remove the absolute value by changing the problem to an OR inequality, the first inequality looking just like the problem with no absolute value, the second flipping the inequality symbol and changing the value to a negative. So $|x| > 2$ beomces $x > 2$ or $x < -2$, as the graph above illustrates.

For all absoloute value inequalities we can also express our answers in interval notation which is done the same way it is done for standard compound inequalities.

We can solve absolute value inequalities much like we solved absolute value equations. Our first step will be to isolate the absolute value. Next we will will remove the absolute value by making a three part inequality if the absolute value is less than a number, or making an OR inequality if the absolute value is greater than a number. Then we will solve these inequalitites. Remember, if we multiply or divide by a negative the inequality symbol will switch directions!

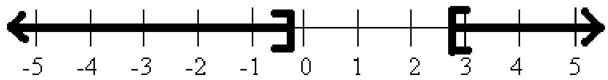
Example 10.

Solve, graph, and give interval notation for the solution

$$\begin{array}{rl}
|4x-5| \geqslant 6 & \text{Absolute value is greater, use OR} \\
4x-5 \geqslant 6 \;\text{ OR }\; 4x-5 \leqslant -6 & \text{Solve} \\
\underline{+5+5} \qquad\quad \underline{+5 \quad +5} & \text{Add 5 to both sides}
\end{array}$$

$$\frac{4x}{4} \geqslant \frac{11}{4} \quad \text{OR} \quad \frac{4x}{4} \leqslant \frac{-1}{4} \qquad \text{Divide both sides by } 4$$

$$x \geqslant \frac{11}{4} \quad \text{OR} \quad x \leqslant -\frac{1}{4} \qquad \text{Graph}$$

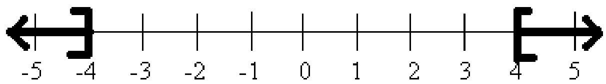


$$\left(-\infty, -\frac{1}{4}\right] \cup \left[\frac{11}{4}, \infty\right) \qquad \text{Interval notation}$$

Example 11.

Solve, graph, and give interval notation for the solution

$$\begin{aligned}
-4 - 3|x| &\leqslant -16 && \text{Add 4 to both sides} \\
\underline{+4 \qquad\quad} & \underline{\quad\; +4} && \\
-3|x| &\leqslant -12 && \text{Divide both sides by } -3 \\
\overline{-3} \qquad & \quad\;\; \overline{-3} && \text{Dividing by } a \text{ negative switches the symbol} \\
|x| &\geqslant 4 && \text{Absolute value is greater, use OR} \\
x \geqslant 4 \;\; \text{OR} \;\; x &\leqslant -4 && \text{Graph}
\end{aligned}$$



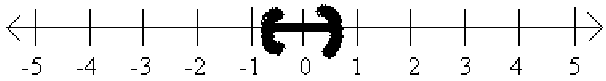
$$(-\infty, -4] \cup [4, \infty) \qquad \text{Interval Notation}$$

In the previous example, we cannot combine -4 and -3 because they are not like terms, the -3 has an absolute value attached. So we must first clear the -4 by adding 4, then divide by -3. The next example is similar.

Example 12.

Solve, graph, and give interval notation for the solution

$$\begin{aligned}
9 - 2|4x+1| &> 3 && \text{Subtract 9 from both sides} \\
\underline{-9 \qquad\qquad} & \underline{\quad -9} && \\
-2|4x+1| &> -6 && \text{Divide both sides by } -2 \\
\overline{\quad -2 \quad} \quad & \quad \overline{-2} && \text{Dividing by negative switches the symbol} \\
|4x+1| &< 3 && \text{Absolute value is less, use three part} \\
-3 < 4x+1 &< 3 && \text{Solve} \\
\underline{-1 \qquad -1} & \underline{-1} && \text{Subtract 1 from all three parts} \\
-3 < 4x &< 2 && \text{Divide all three parts by 4} \\
\overline{4} \quad \overline{4} \; & \; \overline{4} && \\
-\frac{3}{4} < x &< \frac{1}{2} && \text{Graph}
\end{aligned}$$



$$\left(-\frac{3}{4},\frac{1}{2}\right)\quad \text{Interval Notation}$$

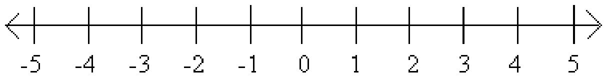
In the previous example, we cannot distribute the -2 into the absolute value. We can never distribute or combine things outside the absolute value with what is inside the absolute value. Our only way to solve is to first isolate the absolute value by clearing the values around it, then either make a compound inequality (and OR or a three part) to solve.

It is important to remember as we are solving these equations, the absolute value is always positive. If we end up with an absolute value is less than a negative number, then we will have no solution because absolute value will always be positive, greater than a negative. Similarly, if absolute value is greater than a negative, this will always happen. Here the answer will be all real numbers.

Example 13.

Solve, graph, and give interval notation for the solution

$$\begin{array}{rl}
12+4|6x-1|<4 & \text{Subtract 12 from both sides}\\
\underline{-12 \qquad\qquad -12} & \\
4|6x-1|<-8 & \text{Divide both sides by 4}\\
\overline{\quad 4 \quad}\qquad \overline{\;4\;} & \\
|6x-1|<-2 & \text{Absolute value can't be less than a negative}
\end{array}$$

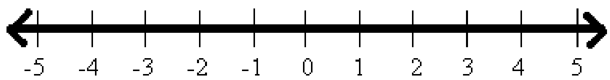


$$\text{No Solution or } \varnothing$$

Example 14.

Solve, graph, and give interval notation for the solution

$$\begin{array}{rl}
5-6|x+7|\leqslant 17 & \text{Subtract 5 from both sides}\\
\underline{-5 \qquad\qquad -5} & \\
-6|x+7|\leqslant 12 & \text{Divide both sides by } -6\\
\overline{\quad -6 \quad}\qquad \overline{-6} & \text{Dividing by a negative flips the symbol}\\
|x+7|\geqslant -2 & \text{Absolute value always greater than negative}
\end{array}$$



$$\text{All Real Numbers or } \mathbb{R}$$

Practice - Absolute Value Inequalities

Solve each inequality graph its solution and give internal notation.

1) $|n| \leqslant -11$

3) $|b| \leqslant -10$

5) $|x| > 5$

7) $10|n| > 30$

9) $-3|x| < 36$

11) $|n| + 4 > -5$

13) $10 - 8|p| \geqslant 18$

15) $9|n| - 3 \geqslant 42$

17) $\left|\frac{m}{9}\right| \geqslant -5$

19) $|9 + x| > -2$

21) $\left|\frac{v+7}{3}\right| \geqslant 5$

23) $7|-7x| \geqslant 98$

25) $-5 + |-8k| \geqslant 51$

27) $8 - 4\left|\frac{x}{9}\right| > 12$

29) $7|-9 + m| + 3 \geqslant 66$

31) $|3n + 10| \leqslant -26$

33) $|10b + 10| > 70$

35) $|-10 + x| \geqslant 8$

37) $|-10 + a| - 3 \geqslant 7$

39) $|3x - 1| - 9 \leqslant -8$

41) $-8|8n - 1| + 4 \geqslant -116$

43) $-10 + 9|3p - 9| < -37$

45) $|9|2 - 10n| - 8 > 100$

2) $|x| \leqslant 7$

4) $|v| \leqslant 2$

6) $10|a| \geqslant 30$

8) $8|k| \leqslant -56$

10) $7|x| \geqslant 28$

12) $10 + 8|k| \geqslant 34$

14) $10|x| + 5 \geqslant 45$

16) $\left|\frac{r}{5}\right| > 2$

18) $|n - 6| > 11$

20) $|b + 8| \geqslant 9$

22) $\frac{|x+1|}{10} \geqslant 4$

24) $-7 + |-5a| > 8$

26) $4 + 6\left|\frac{p}{2}\right| \geqslant 13$

28) $|n - 3| + 4 \geqslant 15$

30) $-10|-3 + r| + 2 \geqslant -18$

32) $|6x + 10| \leqslant 28$

34) $|8v + 1| \geqslant 23$

36) $-3 + |-6n + 1| \geqslant -74$

38) $|6 + 3k| - 4 > 14$

40) $|10x + 4| - 7 < 39$

42) $4|-1 - 9k| + 7 \leqslant -33$

44) $5|-x - 9| - 10 \geqslant 5$

Answers - Graphing and Solving Inequalities

1) $(-5, \infty)$

2) $(-\infty, -2]$

3) $(-\infty, 5]$

4) $(-\infty, 1]$

5) $(-\infty, 5]$

6) $(-5, \infty)$

7) $m < -2$

8) $m \leqslant 1$

9) $x \geqslant 5$

10) $a \leqslant -5$

11) $b > -2$

12) $x > 1$

13) $x \geqslant 110: [110, \infty)$

14) $n \geqslant -26: [-26, \infty)$

15) $r < 1: (-\infty, 1)$

16) $m \leqslant -6: (-\infty, -6]$

17) $n \geqslant -6: [-6, \infty)$

18) $x < 6: (-\infty, 6)$

19) $a < 12: (-\infty, 12)$

20) $v \geqslant 1: [1, \infty)$

21) $x \geqslant 11: [11, \infty)$

22) $x \leqslant -18: (-\infty, -18]$

23) $x > 19: (19, \infty)$

24) $n \leqslant -10: (-\infty, -10]$

25) $p < -1: (-\infty, -1)$

26) $x \leqslant 20: (-\infty, 20]$

27) $m \geqslant 2: [2, \infty)$

28) $n \leqslant 5: (-\infty, 5]$

29) $r > 8: (8, \infty)$

30) $x \leqslant -3: (-\infty, -3]$

31) $b > 1: (1, \infty)$

32) $n \geqslant 0: [0, \infty)$

33) $v < 0: (-\infty, 0)$

34) $x > 2: (2, \infty)$

35) No solution: $\oslash$

36) $n > 1: (1, \infty)$

37) $\{$All real numbers.$\}$: $\mathbb{R}$

38) $p \leqslant 3: (-\infty, 3]$

Answers - Compound Inequalities

1) $n \leqslant -9 \text{ or } n \geqslant 2: (-\infty, -9] \bigcup [2, \infty)$

2) $m \geqslant -4 \text{ or } m < -5 : (-\infty, -5) \bigcup (-4, \infty)$

3) $x \geqslant 5 \text{ or } x < -5: (-\infty, -5) \bigcup [5, \infty)$

4) $r > 0 \text{ or } r < -7 : (-\infty - 7), \bigcup (0, \infty)$

5) $x < -7: (-\infty, -7)$

6) $n < -7 \text{ or } n > 8 : (-\infty - 7), \bigcup (8, \infty)$

7) $-8 < v < 3: (-8, 3)$

8) $-7 < x < 4: (-7, 4)$

9) $b < 5 : (-\infty, 5)$

10) $-2 \leqslant n \leqslant 6 : [-2, 6]$

11) $-7 \leqslant a \leqslant 6 : [-7, 6]$

12) $v \geqslant 6 : [6, \infty)$

13) $-6 \leqslant x \leqslant -2 : [-6, -2]$

14) $-9 \leqslant x \leqslant 0 : [-9, 0]$

15) $3 < k \leqslant 4 : (3, 4]$

16) $-2 \leqslant n \leqslant 4 : [-2, 4]$

17) $-2 < x < 2 : (-2, 2)$

18) $\text{No solution} : \oslash$

19) $-1 \leqslant m < 4 : [-1, 4)$

20) $r > 8 \text{ or } r < -6 : : (-\infty, -6) \bigcup (8, \infty)$

21) $\text{No solution} : \oslash$

22) $x \leqslant 0 \text{ or } x > 8 : (-\infty, 0] \bigcup (8, \infty)$

23) $\text{No solution} : \oslash$

24) $n \geqslant 5 \text{ or } n < 1 : (-\infty, 1(\bigcup [5, \infty)$

25) $5 \leqslant x < 19 : [5, 19)$

26) $\text{No solution} : \oslash$

27) $1 \leqslant v \leqslant 8 : [1, 8]$

28) $a \leqslant 1 \text{ or } a \geqslant 19 : (-\infty, 1] \bigcup [19, \infty)$

29) $k \geqslant 2 \text{ or } k \geqslant -20 : (-\infty, -20) \bigcup [2, \infty)$

30) $-15 \leqslant p < -1 : [-15, -1]$

31) $-1 < x \leqslant 1 : (-1, 1]$

32) $m > 4 \text{ or } m \leqslant -1 : (-\infty, -1] \bigcup (4, \infty)$

33) $n < -14 \text{ or } n \geqslant 17 : (-\infty, -14) \bigcup [17, \infty)$

34) {All real numbers.} : $\mathbb{R}$

Answers - Absolute Value Inequalities

1) No Solution: $\oslash$

2) No Solution: $\oslash$

3) No Solution: $\oslash$

4) $-2 \leqslant v \leqslant 2 : [-2, 2]$

5) $x > 5$ or $x < -5$: $(-\infty, -5) \bigcup (5, \infty)$

6) $a \geqslant 3$ or $a \leqslant -3$: $(-\infty, -3] \bigcup [3, \infty)$

7) $n > 3$ or $n < -3$: $(-\infty, -3) \bigcup (3, \infty)$

8) No Solution: $\oslash$

9) {All real numbers.} : $\mathbb{R}$

10) $x \geqslant 4$ or $x \leqslant -4$: $(-\infty, -4] \bigcup [4, \infty)$

11) {All real numbers.} : $\mathbb{R}$

12) $k \geqslant 3$ or $k \leqslant -3$: $(-\infty, -3] \bigcup [3, \infty)$

13) $\mathrm{No\,Solution} : \oslash$

14) $x \geqslant 4$ or $x \leqslant -4$: $(-\infty, -4] \bigcup [4, \infty)$

15) $n \geqslant 5$ or $n \leqslant -5$: $(-\infty, -5] \bigcup (5, \infty)$

16) $r > 10$ or $r < -10$: $(-\infty, -10) \bigcup (10, \infty)$

17) {All real numbers.} : $\mathbb{R}$

18) $n > 17$ or $n < -5$: $(-\infty, -5) \bigcup (17, \infty)$

19) {All real numbers.} : $\mathbb{R}$

20) $b \geqslant 1$ or $b, = -17$: $(-\infty, -17] \bigcup [1, \infty)$

21) $v \geqslant 8$ or $v \leqslant -22 : (-\infty, -22] \bigcup [8, \infty)$

22) $x \geqslant 39$ or $x \leqslant -41$: $(-\infty, -41] \bigcup [39, \infty)$

23) $x \geqslant -2$ or $x \geqslant 2$: $(-\infty, -2] \bigcup [2, \infty)$

24) $a < -3$ or $a > 3$: $(-\infty, -3) \bigcup (3, \infty)$

25) $k \geqslant -7$ or $k \geqslant 7$: $(-\infty, -7] \bigcup [7, \infty)$

26) $p \geqslant 3$ or $p \leqslant -3$: $(-\infty, -3] \bigcup [3, \infty)$

27) No Solution: $\oslash$

28) $n \geqslant 14$ or $n \leqslant -8$: $(-\infty, -8] \bigcup [14, \infty)$

29) $m \geqslant 18$ or $m \leqslant 0$: $(-\infty, 0] \bigcup [18, \infty)$

30) $1 \leqslant r \leqslant 5$: $[1, 5]$

31) No Solution: $\oslash$

32) $-\frac{19}{3} \leqslant x \leqslant 3$: $[-\frac{19}{3}, 3]$

33) $b > 6 \text{ or } b < -8 \, : \, (-\infty, -8) \bigcup \, (6, \infty)$

34) $v \geqslant \frac{11}{4} \text{ or } v \leqslant -3 \, : \, (-\infty, -3] \bigcup \, [\frac{11}{4}, \infty)$

35) $x \geqslant 18 \text{ or } x \leqslant 2 \, : \, (-\infty, 2] \bigcup \, [18, \infty)$

36) $\{\text{All real numbers.}\} \, : \, \mathbb{R}$

37) $a \geqslant 20 \text{ or } a \leqslant 0 \, : \, (-\infty, 0] \bigcup \, [20, \infty)$

38) $k > 4 \text{ or } k < -8 \, : \, (-\infty, -8) \bigcup \, (4, \infty)$

39) $0 \leqslant x \leqslant \frac{2}{3} : [0, \frac{2}{3}]$

40) $-5 < x < \frac{21}{5} : (-5, \frac{21}{5})$

41) $-\frac{7}{4} \leqslant n \leqslant 2 \, : \, [-\frac{7}{4}, 2]$

42) No Solution: $\oslash$

43) No Solution: $\oslash$

44) $x \leqslant -12 \text{ or } x \geqslant -6 \, : \, (-\infty, -12] \bigcup \, [-6, \infty)$

45) $n < -1 \text{ or } n > \frac{7}{5} \, : \, (-\infty, -1) \bigcup \, (\frac{7}{5}, \infty)$