Beginning and Intermediate Algebra

Chapter 1: Linear Equations

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Chapter 1: Linear Equations

1.1

Solving Linear Equations - One Step Equations

Solving linear equations is an important and fundamental skill in algebra. In algebra, we are often presented with a problem where the answer is known, but part of the problem is missing. The missing part of the problem is what we seek to find. An example of such a problem is shown below.

Example 1.

$$4x + 16 = -4$$

Notice the above problem has a missing part, or unknown, that is marked by x. If we are given that the solution to this equation is -5, it could be plugged into the equation, replacing the x with -5. This is shown in Example 2.

Example 2.

$$\begin{aligned} 4(-5) + 16 &= -4 && \text{Multiply } 4(-5) \\ -20 + 16 &= -4 && \text{Add } -20 + 16 \\ -4 &= -4 && \text{True!} \end{aligned}$$

Now the equation comes out to a true statement! Notice also that if another number, for example, 3, was plugged in, we would not get a true statement as seen in Example 3.

Example 3.

$$\begin{aligned} 4(3) + 16 &= -4 && \text{Multiply } 4(3) \\ 12 + 16 &= -4 && \text{Add } 12 + 16 \\ 28 &\neq -4 && \text{False!} \end{aligned}$$

Due to the fact that this is not a true statement, this demonstates that 3 is not the solution. However, depending on the complexity of the problem, this "guess and check" method is not very efficient. Thus, we take a more algebraic approach to solving equations. Here we will focus on what are called "one-step equations" or equations that only require one step to solve. While these equations often seem very fundamental, it is important to master the pattern for solving these problems so we can solve more complex problems.

Addition Problems

To solve equations, the general rule is to do the opposite. For example, consider Example 4.

Example 4.

$$x + 7 = -5 \qquad \text{The 7 is added to the } x$$

$$\begin{array}{rl} \underline{\mathbf{-7 \quad -7}} & \text{Subtract 7 from both sides to get rid of it} \\ x = -12 & \text{Our solution!} \end{array}$$

Then we get our solution, $x = -12$. The same process is used in each of the following examples.

Example 5.

$4 + x = 8$	$7 = x + 9$	$5 = 8 + x$
$\underline{\mathbf{-4 \quad -4}}$	$\underline{\mathbf{-9 \quad -9}}$	$\underline{\mathbf{-8 - 8}}$
$x = 4$	$-2 = x$	$-3 = x$

Table 1. Addition Examples

Subtraction Problems

In a subtraction problem, we get rid of negative numbers by adding them to both sides of the equation. For example, consider Example 6.

Example 6.

$$\begin{array}{rl} x - 5 = 4 & \text{The 5 is negative, or subtracted from } x \\ \underline{\mathbf{+5 + 5}} & \text{Add 5 to both sides} \\ x = 9 & \text{Our Solution!} \end{array}$$

Then we get our solution $x = 9$. The same process is used in each of the following examples. Notice that each time we are getting rid of a negative number by adding.

Example 7.

$-6 + x = -2$	$-10 = x - 7$	$5 = -8 + x$
$\underline{\mathbf{+6 \qquad +6}}$	$\underline{\mathbf{+7 \quad +7}}$	$\underline{\mathbf{+8 \ +8}}$
$x = 4$	$-3 = x$	$13 = x$

Table 2. Subtraction Examples

Multiplication Problems

With a multiplication problem, we get rid of the number by dividing on both sides. For example consider Example 8.

Example 8.

$$\begin{array}{rl} 4x = 20 & \text{Variable is multiplied by 4} \\ \overline{\mathbf{4}} \quad \overline{\mathbf{4}} & \text{Divide both sides by 4} \\ x = 5 & \text{Our solution!} \end{array}$$

Then we get our solution $x = 5$

With multiplication problems it is very important care is taken with signs. If x is multiplied by a negative then we will divide by a negative. This is shown in example 9.

Example 9.

$$\begin{array}{rl} -5x = 30 & \text{Variable is multiplied by} -5 \\ \overline{\mathbf{-5}} \quad \overline{\mathbf{-5}} & \text{Divide both sides by} -5 \\ x = -6 & \text{Our Solution!} \end{array}$$

The same process is used in each of the following examples. Notice how negative and positive numbers are handeled as each problem is solved.

Example 10.

$8x = -24$	$-4x = -20$	$42 = 7x$
$\overline{\mathbf{8}} \quad \overline{\mathbf{8}}$	$\overline{\mathbf{-4}} \quad \overline{\mathbf{-4}}$	$\overline{\mathbf{7}} \quad \overline{\mathbf{7}}$
$x = -3$	$x = 5$	$6 = x$

Table 3. Multiplication Examples

Division Problems:

In division problems, we get rid of the denominator by multiplying on both sides. For example consider Example 11.

Example 11.

$$\begin{array}{rl} \frac{x}{5} = -3 & \text{Variable is divided by 5} \\ \mathbf{(5)}\frac{x}{5} = -3\mathbf{(5)} & \text{Multiply both sides by 5} \\ x = -15 & \text{Our Solution!} \end{array}$$

Then we get our solution $x = -15$. The same process is used in each of the following examples.

Example 12.

$\frac{x}{-7} = -2$	$\frac{x}{8} = 5$	$\frac{x}{-4} = 9$
$\mathbf{(-7)}\frac{x}{-7} = -2\mathbf{(-7)}$	$\mathbf{(8)}\frac{x}{8} = 5\mathbf{(8)}$	$\mathbf{(-4)}\frac{x}{-4} = 9\mathbf{(-4)}$
$x = 14$	$x = 40$	$x = -36$

Table 4. Division Examples

The process described above is fundamental to solving equations. once this process is mastered, the problems we will see have several more stepts. These problems may seem more complex, but the process and patterns used will remain the same.

Practice - One Step Equations

Solve each equation.

1) $v+9=16$

2) $14=b+3$

3) $x-11=-16$

4) $-14=x-18$

5) $30=a+20$

6) $-1+k=5$

7) $x-7=-26$

8) $-13+p=-19$

9) $13=n-5$

10) $22=16+m$

11) $340=-17x$

12) $4r=-28$

13) $-9=\frac{n}{12}$

14) $\frac{5}{9}=\frac{b}{9}$

15) $20v=-160$

16) $-20x=-80$

17) $340=20n$

18) $\frac{1}{2}=\frac{a}{8}$

19) $16x=320$

20) $\frac{k}{13}=-16$

21) $-16+n=-13$

22) $21=x+5$

23) $p-8=-21$

24) $m-4=-13$

25) $180=12x$

26) $3n=24$

27) $20b=-200$

28) $-17=\frac{x}{12}$

29) $\frac{r}{14}=\frac{5}{14}$

30) $n+8=10$

31) $-7=a+4$

32) $v-16=-30$

33) $10=x-4$

34) $-15=x-16$

35) $13a=-143$

36) $-8k=120$

37) $\frac{p}{20}=-12$

38) $-15=\frac{x}{9}$

39) $9+m=-7$

40) $-19=\frac{n}{20}$

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Linear Equations - Two-Step Equations

After mastering the technique for solving equations that are simple one-step equations, we are ready to consider two-step equations. As we solve two-step equations, the important thing to remember is that everything works backwards! When working with one-step equations, we learned that in order to clear a "plus five" in the equation, we would subtract five from both sides. We learned that to clear "divided by seven" we multiply by seven on both sides. The same pattern applies to the order of operations. When solving for our variable x, we use order of operations backwards as well. This means we will add or subtract first, then multiply or divide second (then exponents, and finally any parentheses or grouping symbols, but that's another lesson). So to solve the equation in Example 1

Example 13.

$$4x - 20 = -8$$

We have two numbers on the same side as the x. We need to move the 4 and the 20 to the other side. We know to move the four we need to divide, and to move the twenty we will add twenty to both sides. If order of operations is doen backwards, we will add or subtract first. Therefore we will add 20 to both sides first. Once we are done with that, we will divide both sides by 4. The steps are shown below.

$$\begin{array}{rl} 4x - 20 = -8 & \text{Start by focusing on the subtract 20} \\ \underline{\mathbf{+20} \ \mathbf{+20}} & \text{Add 20 to both sides} \\ 4x \qquad = 12 & \text{Now we focus on the 4 multiplied by } x \\ \overline{\mathbf{4}} \qquad \overline{\mathbf{4}} & \text{Divide both sides by 4} \\ x = 3 & \text{Our Solution!} \end{array}$$

Notice in Example 2 when we replace the x with 3 we get a true statement.

$$\begin{array}{rl} 4(3) - 20 = -8 & \text{Multiply } 4(3) \\ 12 - 20 = -8 & \text{Subtract } 12 - 20 \end{array}$$

$$-8=-8 \quad \text{True!}$$

The same process is used to solve any two-step equations. Add or subtract first, then multiply or divide. Consider Example 2 and notice how the same process is applied.

Example 14.

$$\begin{array}{rll}
5x+7= & 7 & \text{Start by focusing on the plus 7} \\
\underline{\mathbf{-7} \quad \mathbf{-7}} & & \text{Subtract 7 from both sides} \\
5x = & 0 & \text{Now focus on the multiplication by 5} \\
\overline{\mathbf{5}} \quad \overline{\mathbf{5}} & & \text{Divide both sides by 5} \\
x= & 0 & \text{Our Solution!}
\end{array}$$

Notice in Example 3 the seven subtracted out completely! Many students get stuck on this point, don't forget that we have a number for "nothing left" and that number is zero. With this in mind the process is almost identical to Example 1.

A common error students make with two-step equations is with negative signs. Remember the sign always stays with the number. Consider Example 3

Example 15.

$$\begin{array}{rl}
4-2x=10 & \text{Start by focusing on the positive 4} \\
\underline{\mathbf{-4} \qquad \mathbf{-4}} & \text{Subtract 4 from both sides} \\
-2x=6 & \text{Negative (subtraction) stays on the } 2x \\
\overline{\mathbf{-2}} \quad \overline{\mathbf{-2}} & \text{Divide by } -2 \\
x=-3 & \text{Our Soultion!}
\end{array}$$

The same is true even if there is no coefficient in front of the variable. Consider Example 4.

Example 16.

$$\begin{array}{rl}
8-x=2 & \text{Start by focusing on the positive 8} \\
\underline{\mathbf{-8} \qquad \mathbf{-8}} & \text{Subtract 8 from both sides}
\end{array}$$

$$\begin{array}{rl} -x=-6 & \text{Negative (subtraction) stays on the } x \\ -1x=-6 & \text{Remember, no number in front of variable means 1} \\ \overline{\mathbf{-1}}\quad\overline{\mathbf{-1}} & \text{Divide both sides by } -1 \\ x=6 & \text{Our Solution!} \end{array}$$

Solving two-step equations is a very important skill to master, as we study algebra. The first step is to add or subtract, the second is to multiply or divide. This pattern is seen in each of the following examples.

Example 17.

$-3x+7=-8$ $\underline{\mathbf{-7\ \ -7}}$ $-3x=-15$ $\overline{\mathbf{-3}}\ \overline{\mathbf{-3}}$ $x=5$	$-2+9x=7$ $\underline{\mathbf{+2\quad +2}}$ $9x=9$ $\overline{\mathbf{9}}\ \overline{\mathbf{9}}$ $x=1$	$8=2x+10$ $\underline{\mathbf{-10\quad -10}}$ $-2=2x$ $\overline{\mathbf{2}}\ \overline{\mathbf{2}}$ $-1=x$
$7-5x=17$ $\underline{\mathbf{-7\quad -7}}$ $-5x=10$ $\overline{\mathbf{-5}}\ \overline{\mathbf{-5}}$ $x=-2$	$-5-3x=-5$ $\underline{\mathbf{+5\quad +5}}$ $-3x=0$ $\overline{\mathbf{-3}}\ \overline{\mathbf{-3}}$ $x=0$	$-3=\frac{x}{5}-4$ $\underline{\mathbf{+4\quad +4}}$ $(\mathbf{5})(-1)=\frac{x}{5}(\mathbf{5})$ $-5=x$

Table 5. Two-Step Equation Examples

As problems in algebra become more complex the process covered here will remain the same. In fact, as we solve problems like those in Example 7, each one of them will have several steps to solve, but the last two steps are a two-step equation like we are solving here. This is why it is very important to master two-step equations now!

Example 18.

$$3x^2+4-x+6 \qquad \frac{1}{x-8}+\frac{1}{x}=\frac{1}{3} \qquad \sqrt{5x-5}+1=x \qquad \log_5(2x-4)=1$$

Practice - Two-Step Problems

Solve each equation.

1) $5+\frac{n}{4}=4$

2) $-2=-2m+12$

3) $102=-7r+4$

4) $27=21-3x$

5) $-8n+3=-77$

6) $-4-b=8$

7) $0=-6v$

8) $-2+\frac{x}{2}=4$

9) $-8=\frac{x}{5}-6$

10) $-5=\frac{a}{4}-1$

11) $0=-7+\frac{k}{2}$

12) $-6=15+3p$

13) $-12+3x=0$

14) $-5m+2=27$

15) $24=2n-8$

16) $-37=8+3x$

17) $2=-12+2r$

18) $-8+\frac{n}{12}=-7$

19) $\frac{b}{3}+7=10$

20) $\frac{x}{1}-8=-8$

21) $152=8n+64$

22) $-11=-8+\frac{v}{2}$

23) $-16=8a+64$

24) $-2x-3=-29$

25) $56+8k=64$

26) $-4-3n=-16$

27) $-2x+4=22$

28) $67=5m-8$

29) $-20=4p+4$

30) $9=8+\frac{x}{6}$

31) $-5=3+\frac{n}{2}$

32) $\frac{m}{4}-1=-2$

33) $\frac{r}{8}-6=-5$

34) $-80=4x-28$

35) $-40=4n-32$

36) $33=3b+3$

37) $87=3-7v$

38) $3x-3=-3$

39) $-x+1=-11$

40) $4+\frac{a}{3}=1$

1.3

Solving Linear Equations - General Equations

Often as we are solving linear equations we will need to do some work to set them up into a form we are familiar with solving. This section will focus on manipulating an equation we are asked to solve in such a way that we can use our pattern for solving two-step equations to ultimately arrive at the solution.

One such issue that needs to be addressed is parenthesis. Often the parenthesis can get in the way of solving an otherwise easy problem. As you might expect we can get rid of the unwanted parenthesis by using the distributive property. This is shown in Example 1. Notice the first step is distributing, then it is solved like any other two-step equation.

Example 19.

$$\begin{array}{rl}
4(2x-6)=16 & \text{Distribute 4 through parenthesis}\\
8x-24=16 & \text{Focus on the subtraction first}\\
\underline{\mathbf{+24+24}} & \text{Add 24 to both sides}\\
8x=40 & \text{Now focus on the multiply by 8}\\
\overline{\mathbf{8}}\quad\overline{\mathbf{8}} & \text{Divide both sides by 8}\\
x=5 & \text{Our Solution!}
\end{array}$$

Often after we distribute there will be some like terms on one side of the equation. Example 2 shows distributing to clear the parenthesis and then combining like terms next. Notice we only combine like terms on the same side of the equation. Once we have done this, Example 2 solves just like any other two-step equation.

Convention 20.

$$\begin{array}{rl}
3(2x-4)+9=15 & \text{Distribute the 3 through praenthesis}\\
6x-12+9=15 & \text{Combine like terms, }-12+9\\
6x-3=15 & \text{Focus on the subtraction first}\\
\underline{\mathbf{+3\ \ +3}} & \text{Add 3 to both sides}\\
6x=18 & \text{Now focus on the multiply by 6}\\
\overline{\mathbf{6}}\quad\overline{\mathbf{6}} & \text{Divide both sides by 6}\\
x=3 & \text{Our Solution!}
\end{array}$$

A second type of problem that becomes a two-step equation after a bit of work is one where we see the variable on both sides. This is shown in Example 3.

Example 21.

$$4x-6=2x+10$$

Notice here the x is on both the left and right sides of the equation. This can make it difficult to decide which side to work with. We fix this by moving one of the terms with x to the other side, much like we moved a constant term. It doesn't matter which term gets moved, $4x$ or $2x$, however, it would be the author's suggestion to move the smaller term (to avoid negative coefficients). For this reason we begin this problem by clearing the positive $2x$ by subtracting $2x$ from both sides.

$$\begin{array}{rl}
4x - 6 = 2x + 10 & \text{Notice the variable on both sides} \\
\underline{\mathbf{-2x \quad -2x}} & \text{Subtract } 2x \text{ from both sides} \\
2x - 6 = 10 & \text{Focus on the subtraction first} \\
\underline{\mathbf{+6 \; +6}} & \text{Add 6 to both sides} \\
2x = 16 & \text{Focus on the multiplication by 2} \\
\overline{\mathbf{2}} \; \overline{\mathbf{2}} & \text{Divide both sides by 2} \\
x = 8 & \text{Our Solution!}
\end{array}$$

Example 4 shows the check on this solution. Here the solution is plugged into the x on both the left and right sides before simplifying.

Example 22.

$$\begin{array}{rl}
4(8) - 6 = 2(8) + 10 & \text{Multiply } 4(8) \text{ and } 2(8) \text{ first} \\
32 - 6 = 16 + 10 & \text{Add and Subtract} \\
26 = 26 & \text{True!}
\end{array}$$

The next example, Example 5, illustrates the same process with negative coefficients. Notice first the smaller term with the variable is moved to the other side, this time by adding because the coefficient is negative.

Example 23.

$$\begin{array}{rl}
-3x + 9 = 6x - 27 & \text{Notice the variable on both sides, } -3x \text{ is smaller} \\
\underline{\mathbf{+3x \quad +3x}} & \text{Add } 3x \text{ to both sides} \\
9 = 9x - 27 & \text{Focus on the subtraction by 27} \\
\underline{\mathbf{+27 \quad +27}} & \text{Add 27 to both sides} \\
36 = 9x & \text{Focus on the mutiplication by 9} \\
\overline{\mathbf{9}} \; \overline{\mathbf{9}} & \text{Divide both sides by 9} \\
4 = x & \text{Our Solution}
\end{array}$$

Linear equations can become particularly intersting when the two processes are combined. In the following problems we have parenthesis and the variable on both sides. Notice in each of the following examples we distribute, then combine like terms, then move the variable to one side of the equation.

Example 24.

$$\begin{array}{rl}
2(x-5)+3x=x+18 & \text{Distribute the 2 through parenthesis}\\
2x-10+3x=x+18 & \text{Combine like terms } 2x+3x\\
5x-10=x+18 & \text{Notice the variable is on both sides}\\
\underline{\boldsymbol{-x}\qquad\boldsymbol{-x}} & \text{Subtract } x \text{ from both sides}\\
4x-10=18 & \text{Focus on the subtraction of 10}\\
\underline{\mathbf{+10+10}} & \text{Add 10 to both sides}\\
4x=28 & \text{Focus on multiplication by 4}\\
\overline{\mathbf{4}}\quad\overline{\mathbf{4}} & \text{Divide both sides by 4}\\
x=7 & \text{Our Solution}
\end{array}$$

Sometimes we may have to distribute more than once to clear several parenthesis. Remember to combine like terms after you distribute!

Example 25.

$$\begin{array}{rl}
3(4x-5)-4(2x+1)=5 & \text{Distribute 3 and } -4 \text{ through parenthesis}\\
12x-15-8x-4=5 & \text{Combine like terms } 12x-8x \text{ and } -15-4\\
4x-19\quad=5 & \text{Focus on subtraction of 19}\\
\underline{\mathbf{+19+19}} & \text{Add 19 to both sides}\\
4x=24 & \text{Focus on multiplication by 4}\\
\overline{\mathbf{4}}\quad\overline{\mathbf{4}} & \text{Divide both sides by 4}\\
x=6 & \text{Our Solution}
\end{array}$$

This leads to a 5-step process to solve any linear equation. While all five steps aren't always needed, this can serve as a guide to solving equations.

1. Distribute through any parentheses.
2. Combine like terms on each side of the equation.
3. Get the variables on one side by adding or subtracting
4. Solve the remaining 2-step equation (add or subtract then multiply or divide)
5. Check your answer by plugging it back in for x to find a true statement.

The order of these steps is very important. We can see each of these five steps worked through in Example 8.

Example 26.

$$\begin{array}{rl}
4(2x-6)+9=3(x-7)+8x & \text{Distribute 4 and 3 through parenthesis}\\
8x-24+9=3x-21+8x & \text{Combine like terms } -24+9 \text{ and } 3x+8x\\
8x-15=11x-21 & \text{Notice the variable is on both sides}\\
\underline{\boldsymbol{-8x}\qquad\boldsymbol{-8x}} & \text{Subtract } 8x \text{ from both sides}
\end{array}$$

$$\begin{array}{rl} -15 = 3x - 21 & \text{Focus on subtraction of 21} \\ \underline{+\mathbf{21} \quad\quad +\mathbf{21}} & \text{Add 21 to both sides} \\ 6 = 3x & \text{Focus on multiplication by 3} \\ \overline{\mathbf{3}} \quad \overline{\mathbf{3}} & \text{Divide both sides by 3} \\ 2 = x & \text{Our Solution} \end{array}$$

Check:

$$\begin{array}{rl} 4[2(2) - 6] + 9 = 3[(2) - 7] + 8(2) & \text{Plug 2 in for each } x\text{. Multiply inside parenthesis} \\ 4[4 - 6] + 9 = 3[-5] + 8(2) & \text{Finish parentesis on left, multiply on right} \\ 4[-2] + 9 = -15 + 8(2) & \text{Finish multiplication on both sides} \\ -8 + 9 = -15 + 16 & \text{Add} \\ 1 = 1 & \text{True!} \end{array}$$

When we check our solution of $x = 2$ we found a true statement, $1 = 1$. Therefore, we know our solution $x = 2$ is the correct solution for the problem.

There are two special cases that can come up as we are solving these linear equations. The first is illustrated in Examples 9 and 10. Notice we start by distributing and moving the variables all to the same side.

Example 27.

$$\begin{array}{rl} 3(2x - 5) = 6x - 15 & \text{Distribute 3 through parenthesis} \\ 6x - 15 = 6x - 15 & \text{Notice the variable on both sides} \\ \underline{-\boldsymbol{6x} \quad\quad -\boldsymbol{6x}} & \text{Subtract } 6x \text{ from both sides} \\ -15 = -15 & \text{Variable is gone! True!} \end{array}$$

Here the variable subtracted out completely! We are left with a true statement, $-15 = -15$. If the variables subtract out completely and we are left with a true statement, this indicates that the equation is always true, no matter what x is. Thus, for our solution we say **all real numbers** or $\mathbb{R}$.

Example 28.

$$\begin{array}{rl} 2(3x - 5) - 4x = 2x + 7 & \text{Distribute 2 through parenthesis} \\ 6x - 10 - 4x = 2x + 7 & \text{Combine like terms } 6x - 4x \\ 2x - 10 = 2x + 7 & \text{Notice the variable is on both sides} \\ \underline{-\boldsymbol{2x} \quad\quad -\boldsymbol{2x}} & \text{Subtract } 2x \text{ from both sides} \\ -10 \neq 7 & \text{Variable is gone! False!} \end{array}$$

Again, the variable subtracted out completely! However, this time we are left with a false statement, this indicates that the equation is never true, no matter what x is. Thus, for our solution we say **no solution** or $\varnothing$.

Practice - General Linear Equations

Solve each equation.

1) $2-(-3a-8)=1$

2) $2(-3n+8)=-20$

3) $-5(-4+2v)=-50$

4) $2-8(-4+3x)=34$

5) $66=6(6+5x)$

6) $32=2-5(-4n+6)$

7) $0=-8(p-5)$

8) $-55=8+7(k-5)$

9) $-2+2(8x-7)=-16$

10) $-(3-5n)=12$

11) $-21x+12=-6-3x$

12) $-3n-27=-27-3n$

13) $-1-7m=-8m+7$

14) $56p-48=6p+2$

15) $1-12r=29-8r$

16) $4+3x=-12x+4$

17) $20-7b=-12b+30$

18) $-16n+12=39-7n$

19) $-32-24v=34-2v$

20) $17-2x=35-8x$

21) $-2-5(2-4m)=33+5m$

22) $-25-7x=6(2x-1)$

23) $-4n+11=2(1-8n)+3n$

24) $-7(1+b)=-5-5b$

25) $-6v-29=-4v-5(v+1)$

26) $-8(8r-2)=3r+16$

27) $2(4x-4)=-20-4x$

28) $-8n-19=-2(8n-3)+3n$

29) $-a-5(8a-1)=39-7a$

30) $-4+4k=4(8k-8)$

31) $-57=-(-p+1)+2(6+8p)$

32) $16=-5(1-6x)+3(6x+7)$

33) $-2(m-2)+7(m-8)=-67$

34) $7=4(n-7)+5(7n+7)$

35) $50=8(7+7r)-(4r+6)$

36) $-8(6+6x)+4(-3+6x)=-12$

37) $-8(n-7)+3(3n-3)=41$

38) $-76=5(1+3b)+3(3b-3)$

39) $-61=-5(5r-4)+4(3r-4)$

40) $-6(x-8)-4(x-2)=-4$

41) $-2(8n-4)=8(1-n)$

42) $-4(1+a)=2a-8(5+3a)$

43) $-3(-7v+3)+8v=5v-4(1-6v)$

44) $-6(x-3)+5=-2-5(x-5)$

45) $-7(x-2)=-4-6(x-1)$

46) $-(n+8)+n=-8n+2(4n-4)$

47) $-6(8k+4)=-8(6k+3)-2$

48) $-5(x+7)=4(-8x-2)$

49) $-2(1-7p)=8(p-7)$

50) $8(-8n+4)=4(-7n+8)$

1.4

Solving Linear Equations - Fractions

Often when solving linear equations we will need to work with an equation with fraction coefficients. We can solve these problems as we have in the past. This is demonstrated in Example 1.

Example 29.

$$\frac{3}{4}x - \frac{7}{2} = \frac{5}{6} \quad \text{Focus on subtraction}$$

$$\underline{+\mathbf{\frac{7}{2}} \; +\mathbf{\frac{7}{2}}} \quad \text{Add } \frac{7}{2} \text{ to both sides}$$

Notice we will need to get a common denominator to add $\frac{5}{6} + \frac{7}{2}$. Notice we have a common denominator of 6. So we build up the denominator, $\frac{7}{2}\left(\frac{3}{3}\right) = \frac{21}{6}$, and we can now add the fractions:

$$\frac{3}{4}x - \frac{21}{6} = \frac{5}{6} \quad \text{Same problem, with common denominator 6}$$

$$\underline{+\mathbf{\frac{21}{6}} + \mathbf{\frac{21}{6}}} \quad \text{Add } \frac{21}{6} \text{ to both sides}$$

$$\frac{3}{4}x = \frac{26}{6} \quad \text{Reduce } \frac{26}{6} \text{ to } \frac{13}{3}$$

$$\frac{3}{4}x = \frac{13}{3} \quad \text{Focus on multiplication by } \frac{3}{4}$$

We can get rid of $\frac{3}{4}$ by dividing both sides by $\frac{3}{4}$. Dividing by a fraction is the same as multiplying by the reciprocal, so we will multiply both sides by $\frac{4}{3}$.

$$\left(\mathbf{\frac{4}{3}}\right)\frac{3}{4}x = \frac{13}{3}\left(\mathbf{\frac{4}{3}}\right) \quad \text{Multiply by reciprocal}$$

$$x = \frac{52}{9} \quad \text{Our solution!}$$

While this process does help us arrive at the correct solution, the fractions can make the process quite difficult. This is why we have an alternate method for dealing with fractions - clearing fractions. Clearing fractions is nice as it gets rid of the fractions for the majority of the problem. We can easily clear the fractions by finding the LCD and multiplying each term by the LCD. This is shown in Example 2, the same problem as Example 1, but this time we will solve by clearing fractions.

Example 30.

$$\begin{array}{rl}
\frac{3}{4}x - \frac{7}{2} = \frac{5}{6} & \text{LCD} = 12\text{, multiply each term by } 12 \\
\frac{(\mathbf{12})3}{4}x - \frac{(\mathbf{12})7}{2} = \frac{(\mathbf{12})5}{6} & \text{Reduce each 12 with denominators} \\
(\mathbf{3})3x - (\mathbf{6})7 = (\mathbf{2})5 & \text{Multiply out each term} \\
9x - 42 = 10 & \text{Focus on subtraction by } 42 \\
\underline{+\mathbf{42} + \mathbf{42}} & \text{Add 42 to both sides} \\
9x = 52 & \text{Focus on multiplication by } 9 \\
\overline{\mathbf{9}} \quad \overline{\mathbf{9}} & \text{Divide both sides by } 9 \\
x = \frac{52}{9} & \text{Our Solution}
\end{array}$$

The next example illustrates this as well. Notice the 2 isn't a fraction in the origional equation, but to solve it we put the 2 over 1 to make it a fraction.

Example 31.

$$\begin{array}{rl}
\frac{2}{3}x - 2 = \frac{3}{2}x + \frac{1}{6} & \text{LCD} = 6\text{, multiply each term by } 6 \\
\frac{(\mathbf{6})2}{3}x - \frac{(\mathbf{6})2}{1} = \frac{(\mathbf{6})3}{2}x + \frac{(\mathbf{6})1}{6} & \text{Reduce 6 with each denominator} \\
(\mathbf{2})2x - (\mathbf{6})2 = (\mathbf{3})3x + (\mathbf{1})1 & \text{Multiply out each term} \\
4x - 12 = 9x + 1 & \text{Notice variable on both sides} \\
\underline{-\mathbf{4x} \qquad -\mathbf{4x}} & \text{Subtract } 4x \text{ from both sides} \\
-12 = 5x + 1 & \text{Focus on addition of } 1 \\
\underline{-\mathbf{1} \qquad -\mathbf{1}} & \text{Subtract 1 from both sides} \\
-13 = 5x & \text{Focus on multiplication of } 5 \\
\overline{\mathbf{5}} \quad \overline{\mathbf{5}} & \text{Divide both sides by } 5 \\
-\frac{13}{5} = x & \text{Our Solution}
\end{array}$$

We can use this same process if there are parenthesis in the problem. We will first distribute the coefficient in front of the parenthesis, then clear the fractions. This is seen in Example 4.

Example 32.

$$\begin{array}{rl}
\dfrac{3}{2}\left(\dfrac{5}{9}x+\dfrac{4}{27}\right)=3 & \text{Distribute } \dfrac{3}{2} \text{ through parenthesis, reducing if possible}\\
\dfrac{5}{6}x+\dfrac{2}{9}=3 & \text{LCD} = 18\text{, multiply each term by } 18\\
\dfrac{(\mathbf{18})5}{6}x+\dfrac{(\mathbf{18})2}{9}=\dfrac{(\mathbf{18})3}{9} & \text{Reduce } 18 \text{ with each denominator}\\
(\mathbf{3})5x+(\mathbf{2})2=(\mathbf{18})3 & \text{Multiply out each term}\\
15x+4=54 & \text{Focus on addition of } 4\\
\underline{-\mathbf{4}\quad-\mathbf{4}} & \text{Subtract } 4 \text{ from both sides}\\
15x=50 & \text{Focus on multiplication by } 15\\
.\ \overline{\mathbf{15}}\quad\overline{\mathbf{15}} & \text{Divide both sides by } 15\text{. Reduce on right side.}\\
x=\dfrac{10}{3} & \text{Our Solution}
\end{array}$$

While the problem can take many different forms, the pattern to clear the fraction is the same, after distributing through any parentheses we multiply each term by the LCD and reduce. This will give us a problem with no fractions that is much easier to solve. Example 5 again illustrates this process.

Example 33.

$$\begin{array}{rl}
\dfrac{3}{4}x-\dfrac{1}{2}=\dfrac{1}{3}(\dfrac{3}{4}x+6)-\dfrac{7}{2} & \text{Distribute } \dfrac{1}{3}\text{, reduce if possible}\\
\dfrac{3}{4}x-\dfrac{1}{2}=\dfrac{1}{4}x+2-\dfrac{7}{2} & \text{LCD} = 4\text{, multily each term by } 4.\\
\dfrac{(\mathbf{4})3}{4}x-\dfrac{(\mathbf{4})1}{2}=\dfrac{(\mathbf{4})1}{4}x+\dfrac{(\mathbf{4})2}{1}-\dfrac{(\mathbf{4})7}{2} & \text{Reduce } 4 \text{ with each denominator}\\
(\mathbf{1})3x-(\mathbf{2})1=(\mathbf{1})1x+(\mathbf{4})2-(\mathbf{2})7 & \text{Multiply out each term}\\
3x-2=x+8-14 & \text{Combine like terms } 8-14\\
3x-2=x-6 & \text{Notice variable on both sides}\\
\underline{-\boldsymbol{x}\qquad-\boldsymbol{x}} & \text{Subtract } x \text{ from both sides}\\
2x-2=-6 & \text{Focus on subtraction by } 2\\
\underline{+\mathbf{2}\quad+\mathbf{2}} & \text{Add } 2 \text{ to both sides}\\
2x=-4 & \text{Focus on multilication by } 2\\
\overline{\mathbf{2}}\quad\overline{\mathbf{2}} & \text{Divide both sides by } 2\\
x=-2 & \text{Our Solution}
\end{array}$$

Practice - Fractions

Solve each equation.

1) $\frac{3}{5}(1+p)=\frac{21}{20}$

2) $-\frac{1}{2}=\frac{3}{2}k+\frac{3}{2}$

3) $0=-\frac{5}{4}(x-\frac{6}{5})$

4) $\frac{3}{2}n-\frac{8}{3}=-\frac{29}{12}$

5) $\frac{3}{4}-\frac{5}{4}m=\frac{113}{24}$

6) $\frac{11}{4}+\frac{3}{4}r=\frac{163}{32}$

7) $\frac{635}{72}=-\frac{5}{2}(-\frac{11}{4}+x)$

8) $-\frac{16}{9}=-\frac{4}{3}(\frac{5}{3}+n)$

9) $2b+\frac{9}{5}=-\frac{11}{5}$

10) $\frac{3}{2}-\frac{7}{4}v=-\frac{9}{8}$

11) $\frac{3}{2}(\frac{7}{3}n+1)=\frac{3}{2}$

12) $\frac{41}{9}=\frac{5}{2}(x+\frac{2}{3})-\frac{1}{3}x$

13) $-a-\frac{5}{4}(-\frac{8}{3}a+1)=-\frac{19}{4}$

14) $\frac{1}{3}(-\frac{7}{4}k+1)-\frac{10}{3}k=-\frac{13}{8}$

15) $\frac{55}{6}=-\frac{5}{2}(\frac{3}{2}p-\frac{5}{3}$

16) $-\frac{1}{2}(\frac{2}{3}x-\frac{3}{4})-\frac{7}{2}x=-\frac{83}{24}$

17) $\frac{16}{9}=-\frac{4}{3}(-\frac{4}{3}n-\frac{4}{3})$

18) $\frac{2}{3}(m+\frac{9}{4})-\frac{10}{3}=-\frac{53}{18}$

19) $-\frac{5}{8}=\frac{5}{4}(r-\frac{3}{2})$

20) $\frac{1}{12}=\frac{4}{3}x+\frac{5}{3}(x-\frac{7}{4})$

21) $-\frac{11}{3}+\frac{3}{2}b=\frac{5}{2}(b-\frac{5}{3})$

22) $\frac{7}{6}-\frac{4}{3}n=-\frac{3}{2}n+2(n+\frac{3}{2})$

23) $-(-\frac{5}{2}x-\frac{3}{2})=-\frac{3}{2}+x$

24) $-\frac{149}{16}-\frac{11}{3}r=-\frac{7}{4}r-\frac{5}{4}(-\frac{4}{3}r+1)$

25) $\frac{45}{16}+\frac{3}{2}n=-\frac{7}{4}v-\frac{19}{6}$

26) $-\frac{7}{2}(\frac{5}{3}a+\frac{1}{3})=\frac{11}{4}a+\frac{25}{8}$

27) $\frac{3}{2}(v+\frac{3}{2})=-\frac{7}{4}v-\frac{19}{6}$

28) $-\frac{8}{3}-\frac{1}{2}x=-\frac{4}{3}x-\frac{2}{3}(-\frac{13}{4}x+1)$

29) $\frac{47}{9}+\frac{3}{2}x=\frac{5}{3}(\frac{5}{2}x+1)$

30) $\frac{1}{3}n+\frac{29}{6}=2(\frac{4}{3}n+\frac{2}{3})$

1.5

Solving Linear Equations - Formulas

Solving formulas is much like solving general linear equations. The only difference is we will have several varaibles in the problem and we will be attempting to solve for one specific variable. For example, we may have a formula such as $A = \pi r^2 + \pi \mathrm{rs}$ (formula for surface area of a right circular cone) and we may be interested in solving for the varaible s. This means we want to isolate the s so the equation has s on one side, and everything else on the other. So a solution might look like $s = \frac{A - \pi r^2}{\pi s}$. This second equation gives the same information as the first, they are algebraically equivalent, however, one is solved for the area, while the other is solved for s (slant height of the cone). In this section we will discuss how we can move from the first equation to the second.

When solving formulas for a variable we need to focus on the one varaible we are trying to solve for, all the others are treated just like numbers. This is shown in Example 1. Two parallel problems are shown, the first is a normal one-step equation, the second is a formula that we are solving for x

Example 34.

$$\begin{array}{rrl} 3x = 12 & wx = z & \text{In both problems, } x \text{ is multiplied by something} \\ \overline{\mathbf{3}} \quad \overline{\mathbf{3}} & \overline{\boldsymbol{w}} \quad \overline{\boldsymbol{w}} & \text{To isolate the } x \text{ we divide by 3 or } w. \\ x = 4 & x = \dfrac{z}{w} & \text{Our Solution} \end{array}$$

We use the same process to solve $3x = 12$ for x as we use to solve $wx = z$ for x. Because we are solving for x we treat all the other variables the same way we would treat numbers. Thus, to get rid of the multiplication we divided by w. This same idea is seen in Example 2

Example 35.

$$\begin{array}{rll} m + n = p & \text{for } n & \text{Solving for } n\text{, treat all other variables like numbers} \\ \underline{-\boldsymbol{m} \quad -\boldsymbol{m}} & & \text{Subtract } m \text{ from both sides} \\ n = p - m & & \text{Our Solution} \end{array}$$

As p and m are not like terms, they cannot be combined. For this reason we leave the expression as $p - m$. This same one-step process can be used with grouping symbols.

Example 36.

$$
\begin{array}{rcll}
\frac{a(x-y)}{\boldsymbol{(x-y)}} & = & \frac{b}{\boldsymbol{(x-y)}} \quad \text{for } a & \text{Solving for } a\text{, treat } (x-y) \text{ like } a \text{ number} \\
 & & & \text{Divide both sides by } (x-y) \\
a = \frac{b}{x-y} & & & \text{Our Solution}
\end{array}
$$

Because $(x - y)$ is in parenthesis, if we aren't searching for what is inside the parenthesis, we can keep them together as a group and divide by that group. However, if we are searching for what is inside the parenthesis, we will have to break up the parenthesis by distributing. Example 4 is the same formula, but this time we will solve for x.

Example 37.

$$
\begin{array}{lll}
a(x-y)=b & \text{for } x & \text{Solving for } x\text{, we need to distribute to clear parenthesis} \\
ax-ay=b & & \text{This is } a \text{ two} - \text{step equation, } ay \text{ is subtracted from our } x \text{ term} \\
\underline{\boldsymbol{+ay+ay}} & & \text{Add } ay \text{ to both sides} \\
ax=b+ay & & \text{The } x \text{ is multipied by } a \\
\overline{\boldsymbol{a}} \quad \overline{\boldsymbol{a}} & & \text{Divide both sides by } a \\
x=\frac{b+ay}{a} & & \text{Our Solution}
\end{array}
$$

Be very careful as we isolate x that we do not try and cancil the a on top and bottom of the fraction. This is not allowed if there is any adding or subtracting in the fraction. There is no reducing possible in this problem, so our final reduced answer remains $x=\frac{b+ay}{a}$. The next example is another two-step problem

Example 38.

$$
\begin{array}{lll}
y=mx+b & \text{for } m & \text{Solving for } m\text{, focus on addition first} \\
\underline{\boldsymbol{-b} \qquad \boldsymbol{-b}} & & \text{Subtract } b \text{ from both sides} \\
y-b=mx & & m \text{ is multipied by } x. \\
\overline{\boldsymbol{x}} \qquad \overline{\boldsymbol{x}} & & \text{Divide both sides by } x \\
\frac{y-\mathrm{bx}}{x}=m & & \text{Our Solution}
\end{array}
$$

It is important to note that we know we are done with the problem when the variable we are solving for is isolated or alone on one side of the equation and it does not appear anywhere on the other side of the equation.

The next example is also a two-step equation, it is the problem we started with at the beginning of the lession.

Example 39.

$$\begin{array}{rll}
A = \pi r^2 + \pi r s & \text{for } s & \text{Solvign for } s\text{, focus on what is added to the term with } s \\
\underline{-\boldsymbol{\pi r^2} \quad -\boldsymbol{\pi r^2}} & & \text{Subtract } \pi r^2 \text{ from both sides} \\
A - \pi r^2 = \pi r s & & s \text{ is multipied by } \pi r \\
\overline{\quad \boldsymbol{\pi r} \quad} \quad \overline{\boldsymbol{\pi r}} & & \text{Divide both sides by } \pi r \\
\dfrac{A - \pi r^2}{\pi r} = s & & \text{Our Solution}
\end{array}$$

Again, we cannot reduce the πr in the numberator and denominator because of the subtraction in the problem.

Formulas often have fractions in them and can be solved in much the same way we solved with fractions before. First identify the LCD and then multiply each term by the LCD. After we reduce there will be no more fractions in the problem so we can solve like any general equation from there.

Example 40.

$$\begin{array}{rll}
h = \dfrac{2m}{n} & \text{for } m & \text{To clear the fraction we use LCD} = n \\
(\boldsymbol{n})h = \dfrac{(\boldsymbol{n})2m}{n} & & \text{Multiply each term by } n \\
nh = 2m & & \text{Reduce } n \text{ with denominators} \\
\overline{\mathbf{2}} \quad \overline{\mathbf{2}} & & \text{Divide both sides by } 2 \\
\dfrac{nh}{2} = m & & \text{Our Solution}
\end{array}$$

The same pattern can be seen when we have several fractions in our problem.

Example 41.

$$\begin{array}{rll}
\dfrac{a}{b} + \dfrac{c}{b} = e & \text{for } a & \text{To clear the fraction we use LCD} = b \\
\dfrac{(\boldsymbol{b})a}{b} + \dfrac{(\boldsymbol{b})c}{b} = e\,(\boldsymbol{b}) & & \text{Multiply each term by } b \\
a + c = eb & & \text{Reduce } b \text{ with denominators} \\
\underline{-\boldsymbol{c} \quad -\boldsymbol{c}} & & \text{Subtract } c \text{ from both sides} \\
a = eb - c & & \text{Our Solution}
\end{array}$$

Depending on the context of the problem we may find a formula that uses the same letter, one capital, one lowercase. These represent different values and we must be careful not to combine a captial variable with a lower case variable.

Example 42.

$$a = \frac{A}{2-b} \quad \text{for } b \quad \text{Use LCD } (2-b) \text{ as } a \text{ group}$$

$$\mathbf{(2-b)}a = \frac{\mathbf{(2-b)}A}{2-b} \quad \text{Multiply each term by } (2-b)$$

$$\begin{aligned}
(2-b)a &= A && \text{reduce } (2-b) \text{ with denominator}\\
2a - \text{ab} &= A && \text{Distribute through parenthesis}\\
\underline{-\mathbf{2a}} \quad & \underline{-\mathbf{2a}} && \text{Subtract } 2a \text{ from both sides}\\
-\text{ab} &= A - 2a && \text{The } b \text{ is multipied by } -a\\
\overline{-\mathbf{a}} \quad & \overline{-\mathbf{a}} && \text{Divide both sides by } -a\\
b &= \frac{A-2a}{-a} && \text{Our Solution}
\end{aligned}$$

Notice the A and a were not combined as like terms. This is because a formula will often use a captial letter and lower case letter to represent different variables. Often with formulas there is more than one way to solve for a variable. Example 10 solves the same problem in a slightly different manner. After clearing the denominator, we divide by a to move it to the other side, rather than distributing.

Example 43.

$$a = \frac{A}{2-b} \quad \text{for } b \quad \text{Use LCD} = (2-b) \text{ as } a \text{ group}$$

$$\begin{aligned}
\mathbf{(2-b)}a &= \frac{\mathbf{(2-b)}A}{2-b} && \text{Multiply each term by } (2-b)\\
(2-b)a &= A && \text{Reduce } (2-b) \text{ with denominator}\\
\overline{\mathbf{a}} \quad & \overline{\mathbf{a}} && \text{Divide both sides by } a\\
2 - b &= \frac{A}{a} && \text{Focus on the positive } 2\\
\underline{-\mathbf{2}} \quad & \underline{-\mathbf{2}} && \text{Subtract } 2 \text{ from both sides}\\
-b &= \frac{A}{a} - 2 && \text{Still need to clear the negative}\\
\mathbf{(-1)}(-b) &= \mathbf{(-1)}\frac{A}{a} - 2\mathbf{(-1)} && \text{Multiply (or divide) each term by } -1\\
b &= -\frac{A}{a} + 2 && \text{Our Solution}
\end{aligned}$$

Both answers to Examples 9 and 10 are correct, they are just written in a different form because we solved them in different ways. This is very common with formulas, there may be more than one way to solve for a varaible, yet both are equivalent and correct.

Practice - Formulas

Solve each of the following equations for the indicated variable.

1) ab = c for b

2) g = $\frac{h}{i}$ for h

3) $\frac{f}{g}x = b$ for x

4) $p = \frac{3y}{q}$ for y

5) $3x = \frac{a}{b}$ for x

6) $\frac{\mathrm{ym}}{b} = \frac{c}{d}$ for y

7) E = mc^2 for m

8) DS = ds for D

9) V = $\frac{4}{3}\pi r^3$ for π

10) E = $\frac{\mathrm{mv}^2}{2}$ for m

11) a + c = b for c

12) $x - f = g$ for x

13) c = $\frac{4y}{m+n}$ for y

14) $\frac{\mathrm{rs}}{a-3} = k$ for r

15) V = $\frac{\pi \mathrm{Dn}}{12}$ for D

16) F = k$(R - L)$ for k

17) $P = n(p - c)$ for n

18) $S = L + 2B$ for L

19) $T = \frac{D - d}{L}$ for D

20) $I = \frac{E_a - E_q}{R}$ for E_a

21) L = $L_o(1 + \mathrm{at})$ for L_o

22) ax + b = c for x

23) 2m + p = 4m + q for m

24) q = $6(L - p)$ for L

25) $\frac{k - m}{r} = q$ for k

26) $R = \mathrm{aT} + b$ for T

27) $h = \mathrm{vt} - 16t^2$ for v

28) $S = \pi\mathrm{rh} + \pi r^2$ for h

29) $Q_1 = P(Q_2 - Q_1)$ for Q_2

30) L = $\pi(r_1 + r_2) + 2d$ for r_1

31) $R = \frac{\mathrm{kA}(T_1 + T_2)}{d}$ for T_1

32) $P = \frac{V_1(V_2 - V_1)}{g}$ for V_2

33) ax + b = c for x

34) rt = d for r

35) lwh = V for w

36) V = $\frac{\pi r^2 h}{3}$ for h

37) $\frac{1}{a} + b = \frac{c}{a}$ for a

38) $\frac{1}{a} + b = \frac{c}{a}$ for b

39) $\mathrm{at} - \mathrm{bw} = s$ for t

40) $\mathrm{at} - \mathrm{bw} = s$ for w

41) ax + bx = c for x

42) x + 5y = 3 for x

43) x + 5y = 3 for y

44) 3x + 2y = 7 for x

45) 3x + 2y = 7 for y

46) $5a - 7b = 4$ for a

47) $5a - 7b = 4$ for b

48) $4x - 5y = 8$ for x

49) $4x - 5y = 8$ for y

50) $C = \frac{5}{9}(F - 32)$ for F

1.6

Solving Linear Equations - Absolute Value

When solving equations with absolute value we can end up with more than one possible answer. This is because what is in the absolute value can be either negative or positive and we must account for both possibilities when solving equations. This is illustrated in Example 1.

Example 44.

$$\begin{array}{rl} |x| = 7 & \text{Absolute value can be positive or negative} \\ x = 7 \text{ or } x = -7 & \text{Our Solution} \end{array}$$

Notice that we have considered two possibilities, both the positive and negative. Either way, the absolute value of our number will be positive 7.

When we have absolute values in our problem it is important to first isolate the absolute value, then remove the absolute value by considering both the positive and negative solutions. Notice in Examples 2 and 3, all the numbers outside of the absolute value are moved to the other side first before we remove the absolute value bars and consider both positive and negative solutions.

Example 45.

$$\begin{array}{rl} 5 + |x| = 8 & \text{Notice absolute value is not alone} \\ \underline{-\mathbf{5} \qquad -\mathbf{5}} & \text{Subtract 5 from both sides} \end{array}$$

$$
\begin{aligned}
|x| = 3 &\quad \text{Absolute value can be positive or negative} \\
x = 3 \text{ or } x = -3 &\quad \text{Our Solution}
\end{aligned}
$$

Example 46.

$$
\begin{aligned}
-4|x| = -20 &\quad \text{Notice absolute value is not alone} \\
\overline{\mathbf{-4}} \quad \overline{\mathbf{-4}} &\quad \text{Divide both sdies by } -4 \\
|x| = 5 &\quad \text{Absolute value can be positive or negative} \\
x = 5 \text{ or } x = -5 &\quad \text{Our Solution}
\end{aligned}
$$

Notice we never combine what is inside the absolute value with what is outside the absolute value. This is very important as it will often change the final result to an incorrect solution. Example 4 requires two steps to iscolate the absolute value. The idea is the same as a two-step equation, add or subtract, then multiply or divide.

Example 47.

$$
\begin{aligned}
5|x| - 4 = 26 &\quad \text{Notice the absolute value is not alone} \\
\underline{\mathbf{+4} \quad \mathbf{+4}} &\quad \text{Add 4 to both sides} \\
5|x| = 30 &\quad \text{Absolute value still not alone} \\
\overline{\mathbf{5}} \quad \overline{\mathbf{5}} &\quad \text{Divide both sides by 5} \\
|x| = 6 &\quad \text{Absolute value can be positive or negative} \\
x = 6 \text{ or } x = -6 &\quad \text{Our Solution}
\end{aligned}
$$

Again we see the same process, get the absolute value alone first, then consider the positive and negative solutions. Often the absolute value will have more than just a variable in it. In this case we will have to solve the resulting equations when we consider the positive and negative possibilities. This is shown in Example 5.

Example 48.

$$
\begin{aligned}
|2x - 1| = 7 &\quad \text{Absolute value can be positive or negative} \\
2x - 1 = 7 \text{ or } 2x - 1 = -7 &\quad \text{Two equations to solve}
\end{aligned}
$$

Now notice we have two equations to solve, each equation will give us a different solution. Both equations solve like any other two-step equation.

$$\begin{array}{rcl} \begin{aligned} 2x-1&=7\\ \underline{+\mathbf{1}}&\underline{+\mathbf{1}}\\ 2x&=8\\ \overline{\mathbf{2}}\ &\ \overline{\mathbf{2}}\\ x&=4 \end{aligned} & \quad\text{or}\quad & \begin{aligned} 2x-1&=-7\\ \underline{+\mathbf{1}}&\underline{\ +\mathbf{1}}\\ 2x&=-6\\ \overline{2}\ &\ \overline{2}\\ x&=-3 \end{aligned} \end{array}$$

Thus, from Example 5 we have two solutions, $x=4$ or $x=-3$.

Again, it is important to remember that the absolute value must be alone first before we consider the positive and negative possibilities. This is illustrated in Example 6.

Example 49.

$$2-4|2x+3|=-18$$

To get the absolute value alone we first need to get rid of the 2 by subtracting, then divide by -4. Notice we cannot combine the 2 and -4 becuase they are not like terms, the -4 has the absolute value connected to it. Also notice we do not distribute the -4 into the absolute value. This is because the numbers outside cannot be combined with the numbers inside the absolute value. Thus we get the absolute value alone in the following way:

$$\begin{array}{rl} 2-4|2x+3|=-18 & \text{Notice absolute value is not alone}\\ \underline{-\mathbf{2}\qquad\qquad\qquad -\mathbf{2}} & \text{Subtract 2 from both sides}\\ -4|2x+3|=-20 & \text{Absolute value still not alone}\\ \overline{-\mathbf{4}}\qquad\qquad \overline{-\mathbf{4}} & \text{Divide both sides by } -4\\ |2x+3|=5 & \text{Absoloute value can be positive or negative}\\ 2x+3=5 \ \text{ or } \ 2x+3=-5 & \text{Two equations to solve} \end{array}$$

Now we just solve these two remaining equations to find our solutions.

$$\begin{array}{rcl} \begin{aligned} 2x+3&=5\\ \underline{-\mathbf{3}}&\underline{-\mathbf{3}}\\ 2x&=2\\ \overline{\mathbf{2}}\ &\ \overline{\mathbf{2}}\\ x&=1 \end{aligned} & \quad\text{or}\quad & \begin{aligned} 2x+3&=-5\\ \underline{-\mathbf{3}}&\underline{\ -\mathbf{3}}\\ 2x&=-8\\ \overline{\mathbf{2}}\ &\ \overline{\mathbf{2}}\\ x&=-4 \end{aligned} \end{array}$$

We now have our two solutions, $x=1$ and $x=-4$.

As we are solving absolute value equations it is important to be aware of special cases. Remember the result of an absolute value must always be positive. Notice what happens in Example 7.

Example 50.

$$\begin{array}{rl} 7+|2x-5|=4 & \text{Notice absolute value is not alone} \\ \underline{\mathbf{-7 \qquad\qquad -7}} & \text{Subtract 7 from both sides} \\ |2x-5|=-3 & \text{Result of absolute value is negative!} \end{array}$$

Notice the absolute value equals a negative number! This is impossible with absolute value. When this occurs we say there is **no solution** or $\varnothing$.

One other type of absolute value problem is when two absolute values are equal to eachother. We still will consider both the positive and negative result, the difference here will be that we will have to distribute a negative into the second absolute value for the negative possibility.

Example 51.

$$\begin{array}{cl} |2x-7|=|4x+6| & \text{Absolute value can be positive or negative} \\ 2x-7=4x+6 \quad \text{or} \quad 2x-7=-(4x+6) & \text{make second part of second equation negative} \end{array}$$

Notice the first equation is the positive possibility and has no significant difference other than the missing absolute value bars. The second equation considers the negative possibility. For this reason we have a negative in front of the expression which will be distributed through the equation on the first step of solving. So we solve both these equations as follows:

$$\begin{array}{c} 2x-7=4x+6 \\ \underline{\mathbf{-2x \quad -2x}} \\ -7=2x+6 \\ \underline{\mathbf{-6 \qquad -6}} \\ \frac{-13}{\mathbf{2}}=\frac{2x}{\mathbf{2}} \\ \frac{-13}{2}=x \end{array} \qquad \text{or} \qquad \begin{array}{c} 2x-7=-(4x+6) \\ 2x-7=-4x-6 \\ \underline{\mathbf{+4x \quad +4x}} \\ 6x-7=-6 \\ \underline{\mathbf{+7 \quad +7}} \\ \frac{6x}{\mathbf{6}}=\frac{1}{\mathbf{6}} \\ x=\frac{1}{6} \end{array}$$

This gives us our two solutions, $x=\frac{-13}{2}$ or $x=\frac{1}{6}$.

Practice - Absolute Value Equations

Solve each equation.

1) $|m| = -6$

2) $|r| = -4$

3) $|n| = 4$

4) $|x| = 6$

5) $|b| = 7$

6) $\frac{|v|}{3} = 2$

7) $\frac{|x|}{7} = 5$

8) $\frac{|a|}{9} = -4$

9) $-10 + |k| = -15$

10) $-5 + |p| = 5$

11) $10|x| + 7 = 57$

12) $10|n| - 10 = 70$

13) $10 - 5|m| = 70$

14) $-6 - |r| = -11$

15) $9|x| - 4 = 5$

16) $|4 + b| = 4$

17) $\left|\frac{n}{10}\right| = 1$

18) $|x - 3| = 2$

19) $|v + 10| = 2$

20) $|9 - n| = 12$

21) $-4 - |a - 5| = -13$

22) $\frac{|9v|}{6} = 1$

23) $10|-6x| = 60$

24) $\left|\frac{x}{8}\right| + 6 = 7$

25) $-7\left|\frac{n}{7}\right| = -2$

26) $7\left|\frac{k}{7}\right| + 8 = 15$

27) $-8|-7 + p| - 6 = -14$

28) $2|n + 8| - 8 = 28$

29) $-3|7 + x| - 7 = -1$

30) $7|m - 6| - 9 = -72$

31) $|-7 - 5r| = 32$

32) $|-3x - 5| = 14$

33) $|8n - 6| = 66$

34) $|6 - 6b| = 30$

35) $|2v + 7| = 11$

36) $\frac{|-n + 6|}{6} = 0$

37) $9|10 + 6x| = 72$

38) $|2 + 6a| - 9 = 29$

39) $-3 + |6 + 6k| = -45$

40) $|p + | + 5 = 17$

41) $|2n + 5| + 5 = 0$

42) $2 + 3|6 + 5x| = 89$

43) $3 - 2|5 - m| = 9$

44) $-1 + 9|8r - 4| = 35$

45) $|-10x - 4| - 10 = 66$

46) $|5x + 3| = |2x - 1|$

47) $|2 + 3x| = |4 - 2x|$

48) $|3x - 4| = |2x + 3|$

49) $\left|\frac{2x - 5}{3}\right| = \left|\frac{3x + 4}{2}\right|$

50) $\left|\frac{4x - 2}{5}\right| = \left|\frac{6x + 3}{2}\right|$

1.7

Solving Linear Equations - Variation

One application of solving linear equations is variation. Often different events are related by what is called the constant of variation. For example, the time it takes to travel a certain distance is related to how fast you are traveling. The faster you travel, the less time it take to get there. This is one type of variation problem, we will look at three types of variation here. Variation problems have two or three variables and a constant in them. The constant, usually noted with a k, describes the relationship and does not change as the other variables in the problem change. There are two ways to set up a variation problem, the first solves for one of the variables, a second method is to solve for the constant. Here we will use the second method.

The greek letter pi (π) is used to represent the ratio of the circumference of a circle to its diameter. If you take any circle and divide the circumference of the circle by the diameter you will always get the same value, about 3.14159... If you have a bigger circumference you will also have a bigger diameter. This relationship is called **direct variation** or **directly proportional**. If we see this phrase in the problem we know to divide to find the constant of variation.

Example 52.

$$\begin{array}{rl} m\text{ is varies directly as }n & \text{"Directly" tells us to divide} \\ \dfrac{m}{n}=k & \text{Our formula for the relationship} \end{array}$$

In kickboxing, one will find that the longer the board, the easier it is to break. If you multiply the force required to break a board by the length of the board you will also get a constant. Here, we are multiplying the variables, which means as one variable increases, the other variable decreases. This relationship is called **indirect variation** or **inversly proportional**. If we see this phrase in the problem we know to multiply to find the constant of variation.

Example 53.

$$\begin{array}{rl} y\text{ is inversely proportional to }z & \text{"Inversely" tells us to multiply} \\ yz=k & \text{Our formula for the relationship} \end{array}$$

The formula for the area of a triangle has three variables in it. If we divide the area by the base times the height we will also get a constant, $\frac{1}{2}$. This relationship is called **joint variation** or **jointly proportional**. If we see this phrase in the problem we know to divide the first variable by the product of the other two to find the constant of variation.

Example 54.

$$\begin{aligned} A \text{ varies jointly as } x \text{ and } y & \quad \text{"Jointly" tells us to divide by the product} \\ \frac{A}{xy} = k & \quad \text{Our formula for the relationship} \end{aligned}$$

Once we have our formula for the relationship in a variation problem, we use given or known information to calculate the constant of variation. This is shown for each type of variation in Example 4, 5 and 6.

Example 55.

$$w \text{ is directly proportional to } y \text{ and } w = 50 \text{ when } y = 5$$

$$\begin{aligned} \frac{w}{y} = k & \quad \text{"directly" tells us to divide} \\ \frac{(50)}{(5)} = k & \quad \text{Substitute known values} \\ 10 = k & \quad \text{Evaluate to find our constant} \end{aligned}$$

Example 56.

$$c \text{ varies indirectly as } d \text{ and } c = 4.5 \text{ when } d = 6$$

$$\begin{aligned} cd = k & \quad \text{"indirectly" tells us to multiply} \\ (4.5)(6) = k & \quad \text{Substitute known values} \\ 27 = k & \quad \text{Evaluate to find our constant} \end{aligned}$$

Example 57.

$$x \text{ is jointly proportional to } y \text{ and } z \text{ and } x = 48 \text{ when } y = 2 \text{ and } z = 4$$

$$\begin{aligned} \frac{x}{yz} = k & \quad \text{"Jointly" tells us to divide by the product} \\ \frac{(48)}{(2)(4)} = k & \quad \text{Substitute known values} \\ 6 = k & \quad \text{Evaluate to find our constant} \end{aligned}$$

Once we have found the constant of variation we can use it to find other combinations in the same relationship. Each of these problems we solve will have three important steps, none of which should be skipped.

1. Find the formula for the relationship using the type of variation
2. Find the constant of variation using known values
3. Answer the question using the constant of variation

Examples 7, 8, and 9 show how this process is worked out for each type of variation.

Example 58.

The price of an item varies directly with the sales tax. If a \$25 item has a sales tax of \$2, what will the tax be on a \$40 item?

$$\begin{aligned}
\frac{p}{t} &= k && \text{"Directly" tells us to divide price } (p) \text{ and tax } (t)\\
\frac{(25)}{(2)} &= k && \text{Substitute known values for price and tax}\\
12.5 &= k && \text{Evaluate to find our constant}\\
\frac{40}{t} &= 12.5 && \text{Using our constant, substitute 40 for price to find the tax}\\
\frac{(\boldsymbol{t})40}{t} &= 12.5(\boldsymbol{t}) && \text{Multiply by LCD} = t \text{ to clear fraction}\\
40 &= 12.5t && \text{Reduce the } t \text{ with the denominator}\\
\overline{\mathbf{12.5}}\ & \ \overline{\mathbf{12.5}} && \text{Divide by 12.5}\\
3.2 &= t && \text{Our solution: Tax is \$3.20}
\end{aligned}$$

Example 59.

The speed (or rate) Josiah travels to work is inversely proportional to time it takes to get there. If he travels 35 miles per hour it will take him 2.5 hours to get to work. How long will it take him if he travels 55 miles per hour?

$$\begin{aligned}
rt &= k && \text{"Inversely" tells us to multiply the rate and time}\\
(35)(2.5) &= k && \text{Substitute known values for rate and time}\\
87.5 &= k && \text{Evaluate to find our constant}\\
55t &= 87.5 && \text{Using our constant, substitute 55 for rate to find the time}\\
\overline{\mathbf{55}}\ & \ \overline{\mathbf{55}} && \text{Divide both sides by 55}\\
t &\approx 1.59 && \text{Our solution: It takes him 1.59 hours to get to work}
\end{aligned}$$

Example 60.

The amount of simple interest earned on an investment varies jointly as the principle (amount invested) and the time it is invested. In an account, $150 invested for 2 years earned $12 in interest. How much interest would be earned on a $220 investment for 3 years?

$$
\begin{array}{rl}
\dfrac{I}{Pt} = k & \text{"Jointly" divide Interest } (I) \text{ by product of Principle } (P) \,\&\, \text{time } (t)\\
\dfrac{(12)}{(150)(2)} = k & \text{Substitute known values for Interest, Principle and time}\\
0.04 = k & \text{Evaluate to find our constant}\\
\dfrac{I}{(220)(3)} = 0.04 & \text{Using constant, substitute 220 for principle and 3 for time}\\
\dfrac{I}{660} = 0.04 & \text{Evaluate denominator}\\
\dfrac{\mathbf{(660)}I}{660} = 0.04\mathbf{(660)} & \text{Multiply by 660 to isolate the variable}\\
I = 26.4 & \text{Our Solution: The investment earned \$26.40 in interest}
\end{array}
$$

Sometimes a variation problem will ask us to do something to a variable as we set up the formula for the relationship. For example, π can be thought of as the ratio of the area and the radius squared. This is still direct variation, we say the area varies directly as the radius square and thus our variable is squared in our formula. This is shown in Example 10

Example 61.

The area of a circle is directly proportional to the square of the radius. A circle with a radus of 10 has an area of 314. What will the area be on a circle of radius 4?

$$
\begin{array}{rl}
\dfrac{A}{r^2} = k & \text{"Direct" tells us to divide, be sure we use } r^2 \text{ for the denominator}\\
\dfrac{(314)}{(10)^2} = k & \text{Substitute known values into our formula}\\
\dfrac{(314)}{100} = k & \text{Exponents first}\\
3.14 = k & \text{Divide to find our constant}\\
\dfrac{A}{(4)^2} = 3.14 & \text{Using the constant, use 4 for } r\text{, don't forget the squared!}\\
\dfrac{A}{16} = 3.14 & \text{Evaluate the exponent}\\
\dfrac{\mathbf{(16)}A}{16} = 3.14\mathbf{(16)} & \text{Multiply both sides by 16}\\
A = 50.24 & \text{Our Solution: Area is 50.24}
\end{array}
$$

When solving variation problems it is important to take the time to clearly state the variation formula, find the constant, and solve the final equation.

Practice - Variation

Write the formula that expresses the relationship described

1. c varies directly as a
2. x is jointly propotional to y and z
3. w varies inversely as x
4. r varies directly as the square of s
5. f varies jointly as x and y
6. j is inversely proportional to the cube of m
7. h is directly proportional to b
8. x is jointly proportional with the square of a and the square root of b
9. a is inversely proportional to b

Find the constant of variation and write the formula to express the relationship using that constant

10. a varies directly as b and a = 15 when b = 5
11. p is jointly proportional to q and r and p = 12 when q = 8 and r = 3
12. c varies inversely as d adn c = 7 when d = 4
13. t varies directly as the square of u and t = 6 when u = 3
14. e varies jointly as f and g and e = 24 when f = 3 and g = 2
15. w is inversely proportional to the cube of x and w is 54 when x = 3
16. h is directly proportional to j and h = 12 when j = 8
17. a is jointly proportional with the square of x and the square root of y and a = 25 when x = 5 and y = 9
18. m is inversely proportional to n and m = 1.8 when n = 2.1

Solve each of the following variation problems by setting up a formulat to express the relationship, finding the constant, and then answering the question.

19. The electrical current in amperes, in a circuit varies directly as teh voltage. When 15 volts are applied, the current is 5 amperes. what is the current when 18 volts are applied?

20. The current in an electrical conductor varies inversely as the resistance of the conductor. If the current is 12 ampere when the resistance is 240 ohms, what is the current when the resistance is 540 ohms?

21. Hooke's law states that the distance that a spring is stretched by hanging object varies directly as the mass of the object. If the distance is 20 cm when the mass is 3 kg, what is the distance when the mass is 5 kg?

22. The volume of a gas varies inversely as the pressure upon it. The volume of a gas is 200 cm^3 under a pressure of 32 kg/cm^2. What will be its volume under a pressure of 40 kg/cm^2?

23. The number of aluminum cans used each year varies directly as the number of people using the cans. If 250 people use 60,000 cans in one year, how many cans are used each year in Dallas, which has a population of 1,008,000?

24. The time required to do a job varies inversely as the number of peopel working. It takes 5hr for 7 bricklayers to build a park well. How long will it take 10 bricklayers to complete the job?

25. According to Fidelity Investment Vision Magazine, the average weekly allowance of children varies directly as their grade level. In a recent year, the average allowance of a 9th-grade student was 9.66 dollars per week. What was the average weekly allowance of a 4th-grade student?

26. The wavelength of a radio wave varies inversely as its frequency. A wave with a frequency of 1200 kilohertz has a length of 300 meters. What is the length of a wave with a frequency of 800 kilohertz?

27. The number of kilograms of water in a human body varies directly as the mass of the body. A 96-kg person contains 64 kg of water. How many kilo grams of water are in a 60-kg person?

28. The time required to drive a distance of a fixed distance varies inversely as the speed. It takes 5 hr at a speed of 80 km/h to drive a fixed distance. How long will it take to drive the same distance at a speed of 70 km/h?

29. The weight of an object on Mars varies directly as its weight on Earth. A person weighs 95lb on Earth weighs 38 lb on Mars. How much would a 100-lb person weigh on Mars?

30. At a constant temperature, the volume of a gas varies inversely as the pressure. If the pressure of a certain gas is 40 newtons per square meter when the volume is 600 cubic meters what will the pressure be when the volume is reduced by 240 cubic meters?

31. The time required to empty a tank varies inversely as the rate of pumping. If a pump can empty a tank in 45 min at the rate of 600 kL/min, how long will it take the pump to empty the same tank at the rate of 1000 kL/min?

32. Wind resistance, or atmospheric drag, tends to slow down moving objects. Atmospheric drag varies jointly as an object's surface area of 37.8 ft^2 experiences a drag of 222 N (Newtons), how fast must a car with 51 ft^2 of surface area travel in order to experience a dracforce of 430 N?

33. The stopping distance of a car after the brakes have been applied varies directly as the square of the speed r. If a car traveling 60 mph can stop in 200 ft, how fast can a car go and still stop in 72 ft?

34. The drag force on a boat varies jointly as the wetted surface area and the square of the velocity of a boat. If a boat going 6.5 mph experiences a drag force of 86 N when the wetted surface area is 41.2 ft^2, how fast must a boat with 28.5 ft^2 of wetted surface area go in order to experience a drag force of 94N?

35. The intensity of a light from a light bulb varies inversely as the square of the distance from the bulb. suppose intensity is 90 $\mathrm{W/m}^2$ (watts per square meter) when the sitance is 5 m. How much further would it be to a point where the intesity is 40 $\mathrm{W/m}^2$?

36. The volume of a cone varies jointly as its height, and the square of its radius. If a cone with a height of 8 centimeters and a radius of 2 centimeters has a volume of 33.5 cm^3, what is the volume of a cone with a height of 6 centimeters and a radius of 4 centimeters?

37. The intensity of a television signal varies inversely as teh square of the distance from the transmitter. If the intensity is 25 $\mathrm{W/m}^2$ at a distance of 2 km, how far from the trasmitter are you when the intensity is 2.56 $\mathrm{W/m}^2$?

38. The intensity of illumination falling on a surface from a given source of light is inversely proportional to the square of the distance from the souce of light. The unit for measuring the intesity of illumination is usually the footcandle. If a given source of light gives an illumination of 1 foot-candle at a distance of 10 feet, what would the illumination be from the same source at a distance of 20 feet?

39. The weight of an object varies inversely as the sqare of the distance from the center of the earth. At sea level (6400 km from the center of the earch), an astronaut weighs 100 lb. How far above the earth must the astronaut be in order to weigh 64 lb?

1.8

Linear Equations - Word Problems

Word problems can be tricky. Often it takes a bit of practice to convert the english sentence into a mathematical sentence. This is what we will focus on here with some basic number problems, geometry problems, and parts problems.

A few important phrases are described below that can give us clues for how to set up a problem.

- **A number** (or unknown, a value, etc) often becomes our variable
- **Is** (or other forms of is: was, will be, are, etc) often represents equals $(=)$

  x is 5 becomes $x = 5$
- **More than** often represents addition and is usually built backwards, writing the second part plus the first

  Three more than a number becomes $x + 3$
- **Less than** often represents subtraction and is usually built backwards as well, writing the second part minus the first

  Four less than a number becomes $x - 4$

Using these key phrases we can take a number problem and set up and equation and solve.

Example 62.

If 28 less than five times a certain number is 232. What is the number?

$$\begin{array}{rl} 5x - 28 & \text{Subtraction is built backwards, multiply the unknown by 5} \\ 5x - 28 = 232 & \text{Is translates to equals} \\ \underline{\mathbf{+28} \quad \mathbf{+28}} & \text{Add 28 to both sides} \\ 5x = 260 & \text{The variable is multiplied by 5} \\ \overline{\mathbf{5}} \quad \overline{\mathbf{5}} & \text{Divide both sides by 5} \\ x = 52 & \text{The number is 52.} \end{array}$$

This same idea can be extended to a more invovled problem as shown in Example 2.

Example 63.

Fifteen more than three times a number is the same as ten less than six times the number. What is the number

$$\begin{array}{rl}
3x+15 & \text{First, addition is built backwards} \\
6x-10 & \text{Then, subtraction is also built backwards} \\
3x+15=6x-10 & \text{Is between the parts tells us they must be equal} \\
\underline{\mathbf{-3x \quad -3x}} & \text{Subtract } 3x \text{ so variable is all on one side} \\
15=3x-10 & \text{Now we have } a \text{ two} - \text{step equation} \\
\underline{\mathbf{+10 \quad +10}} & \text{Add 10 to both sides} \\
25=3x & \text{The variable is multiplied by 3} \\
\overline{\mathbf{3}} \quad \overline{\mathbf{3}} & \text{Divide both sides by 3} \\
\frac{25}{3}=x & \text{Our number is } \frac{25}{3}
\end{array}$$

Another type of number problem involves consecutive numbers. **Consecutive numbers** are numbers that come one after the other, such as 3, 4, 5. If we are looking for several consecutive numbers it is important to first identify what they look like with variables before we set up the equation. This is shown in Example 3.

Example 64.

The sum of three consecutive integers is 93. What are the integers?

$$\begin{array}{rl}
\text{First } x & \text{Make the first number } x \\
\text{Second } x+1 & \text{To get the next number we go up one or } +1 \\
\text{Third } x+2 & \text{Add another 1 (2 total) to get the third} \\
F+S+T=93 & \text{First } (F) \text{ plus Second } (S) \text{ plus Third } (T) \text{ equals 93} \\
(x)+(x+1)+(x+2)=93 & \text{Replace } F \text{ with } x, S \text{ with } x+1, \text{ and } T \text{ with } x+2 \\
x+x+1+x+2=93 & \text{Here the parenthesis aren}'t \text{ needed.} \\
3x+3=93 & \text{Combine like terms } x+x+x \text{ and } 2+1 \\
\underline{\mathbf{-3 \quad -3}} & \text{Add 3 to both sides} \\
3x=90 & \text{The variable is multiplied by 3} \\
\overline{\mathbf{3}} \quad \overline{\mathbf{3}} & \text{Divide both sides by 3} \\
x=30 & \text{Our solution for } x \\
\text{First } 30 & \text{Replace } x \text{ in our origional list with 30} \\
\text{Second } (30)+1=31 & \text{The numbers are 30, 31, and 32} \\
\text{Third } (30)+2=32 &
\end{array}$$

Sometimes we will work consective even or odd integers, rather than just consecutive integers. When we had consecutive integers, we only had to add 1 to get to the next number so we had x, $x + 1$, and $x + 2$ for our first, second, and third number respectivly. With even or odd numbers they are spaced apart by two. So if we want three consecutive even numbers, if the first is x, the next number would be $x + 2$, then finally add two more to get the third, $x + 4$. The same is true for consecutive odd numbers, if the first is x, the next will be $x + 2$, and the third would be $x + 4$. It is important to note that we are still adding 2 and 4 even when the numbers are odd. This is because the phrase "odd" is refering to our x, not to what is added to the numbers. Consider the next two examples.

Example 65.

The sum of three consecutive even numbers is 246. What are the numbers?

$$\begin{array}{rl}
\text{First } x & \text{Make the first } x \\
\text{Second } x+2 & \text{Even numbers, so we add 2 to get the next} \\
\text{Third } x+4 & \text{Add 2 more (4 total) to get the third} \\
F+S+T=246 & \text{Sum means add First } (F) \text{ plus Second } (S) \text{ plus Third } (T) \\
(x)+(x+2)+(x+4)=246 & \text{Replace each } F, S, \text{ and } T \text{ with what we labeled them} \\
x+x+2+x+4=246 & \text{Here the parenthesis are not needed} \\
3x+6=246 & \text{Combine like terms } x+x+x \text{ and } 2+4 \\
\underline{\mathbf{-6}\quad\mathbf{-6}} & \text{Subtract 6 from both sides} \\
3x=240 & \text{The variable is multiplied by 3} \\
\overline{\mathbf{3}}\quad\overline{\mathbf{3}} & \text{Divide both sides by 3} \\
x=80 & \text{Our solution for } x \\
\text{First } 80 & \text{Replace } x \text{ in the origional list with 80.} \\
\text{Second } (80)+2=82 & \text{The numbers are 80, 82, and 84.} \\
\text{Third } (80)+4=84 &
\end{array}$$

Example 66.

Find three consecutive odd integers so that the sum of twice the first, the second and three times the third is 152.

$$\begin{array}{rl}
\text{First } x & \text{Make the first } x \\
\text{Second } x+2 & \text{Odd numbers so we add 2 (same as even!)} \\
\text{Third } x+4 & \text{Add 2 more (4 total) to get the third} \\
2F+S+3T=152 & \text{Twice the first gives } 2F \text{ and three times the third gives } 3T \\
2(x)+(x+2)+3(x+4)=152 & \text{Replace } F, S, \text{ and } T \text{ with what we labled them} \\
2x+x+2+3x+12=152 & \text{Distirbute through parenthesis} \\
6x+14=152 & \text{Combine like terms } 2x+x+3x \text{ and } 2+14 \\
\underline{\mathbf{-14}\quad\mathbf{-14}} & \text{Subtract 14 from both sides} \\
6x=138 & \text{Variable is multiplied by 6}
\end{array}$$

$$\begin{array}{rl}
\overline{\mathbf{6}} \quad \overline{\mathbf{6}} & \text{Divide both sides by 6} \\
x = 23 & \text{Our solution for } x \\
\text{First } 23 & \text{Replace } x \text{ with 23 in the original list} \\
\text{Second } (23) + 2 = 25 & \text{The numbers are 23, 25, and 27} \\
\text{Third } (23) + 4 = 27 &
\end{array}$$

When we started with our first, second, and third numbers for both even and odd we had x, $x + 2$, and $x + 4$. The numbers added do not change with odd or even, it is our answer for x that will be odd or even.

Another example of translating english sentences to mathematical sentences comes from geometry. A well known property of triangles is that all three angles will always add to 180. For example, the first angle may be 50 degrees, the second 30 degrees, and the third 100 degrees. If you add these together, $50 + 30 + 100 = 180$. We can use this property to find angles of triangles.

Example 67.

The second angle of a triangle is double the first. The third angle is 40 less than the first. Find the three angles.

$$\begin{array}{rl}
\text{First } x & \text{With nothing given about the first we make that } x \\
\text{Second } 2x & \text{The second is double the first,} \\
\text{Third } x - 40 & \text{The third is 40 less than the first} \\
F + S + T = 180 & \text{All three angles add to 180} \\
(x) + (2x) + (x - 40) = 180 & \text{Replace } F, S, \text{ and } T \text{ with the labeled values.} \\
x + 2x + x - 40 = 180 & \text{Here the parenthesis are not needed.} \\
4x - 40 = 180 & \text{Combine like terms, } x + 2x + x \\
\underline{\mathbf{+\,40 \quad +\,40}} & \text{Add 40 to both sides} \\
4x = 220 & \text{The variable is multiplied by 4} \\
\overline{\mathbf{4}} \quad \overline{\mathbf{4}} & \text{Divide both sides by 4} \\
x = 55 & \text{Our solution for } x \\
\text{First } 55 & \text{Replace } x \text{ with 55 in the original list of angles} \\
\text{Second } 2(55) = 110 & \text{Our angles are 55, 110, and 15} \\
\text{Third } (55) - 40 = 15 &
\end{array}$$

Another geometry problem involves perimeter or the distance around an object. For example, consider a rectangle has a length of 8 and a width of 3. Their are two lengths and two widths in a rectangle (opposite sides) so we add $8 + 8 + 3 + 3 = 22$. As there are two lengths and two widths in a rectangle an alternative to find the perimeter of a rectangle is to use the formula $P = 2L + 2W$. So for the rectangle of length 8 and width 3 the formula would give, $P = 2(8) + 2(3) = 16 + 6 = 22$. With problems that we will consider here the formula $P = 2L + 2W$ will be used.

Example 68.

The perimeter of a rectangle is 44. The length is 5 less than double the width. Find the dimensions.

Length x	We will make the length x
Width $2x - 5$	Width is five less than two times the length
$P = 2L + 2W$	The formula for perimeter of a rectangle
$(44) = 2(x) + 2(2x - 5)$	Replace P, L, and W with labeled values
$44 = 2x + 4x - 10$	Distribute through parenthesis
$44 = 6x - 10$	Combine like terms $2x + 4x$
$\underline{+\mathbf{10} \quad +\mathbf{10}}$	Add 10 to both sides
$54 = 6x$	The variable is multiplied by 6
$\overline{\mathbf{6}} \quad \overline{\mathbf{6}}$	Divide both sides by 6
$9 = x$	Our solution for x
Length 9	Replace x with 9 in the origional list of sides
Width $2(9) - 5 = 13$	The dimensions of the rectangle are 9 by 13.

We have seen that it is imortant to start by clearly labeling the variables in a short list before we begin to solve the problem. This is important in all word problems involving variables, not just consective numbers or geometry problems. This is shown in the following example.

Example 69.

A sofa and a love seat together costs \$444. The sofa costs double the love seat. How much do they each cost?

Love Seat x	With no information about the love seat, this is our x
Sofa $2x$	Sofa is double the love seat, so we multiply by 2
$S + L = 444$	Together they cost 444, so we add.
$(x) + (2x) = 444$	Replace S and L with labled values
$3x = 444$	Parenthesis are not needed, combine like terms $x + 2x$
$\overline{\mathbf{3}} \quad \overline{\mathbf{3}}$	Divide both sides by 3
$x = 148$	Our solution for x
Love Seat 148	Replace x with 148 in the origional list
Sofa $2(148) = 296$	The love seat costs \$148 and the sofa costs \$296.

Be careful on problems such as these. Many students see the phrase "double" and believe that means we only have to divide the 444 by 2 and get \$222 for one or both of the prices. As you can see this will not work. By clearly labeling the variables in the original list we know exactly how to set up and solve these problems.

Practice - Word Problems

Solve.

1. When five is added to three more than a certain number, the result is 19. What is the number?

2. If five is subtracted from three times a ce3rtain number, the result is 10. What is the number?

3. When 18 is subtracted from six times a certain number, the result is -42. What is the number?

4. A certain number added twice to itself equals 96. What is the number?

5. A number plus itself, plus twice itself, plus 4 times itself, is equal to -104. What is the number?

6. Sixty more than nine times a number is the same as two less than ten times the number. What is the number?

7. Eleven less tahn seven times a number is five more than six times the number. Find the number.

8. Fourteen less than eight times a number is three more than four times the number. What is the number?

9. The sum of three consectutive integers is 108. What are the integers?

10. The sum of three consecutive integers is -126. What are the integers?

11. Find three consecutive integers such that the sum of the first, twice the second, and three times the third is -76.

12. The sum of two consectutive even integers is 106. What are the integers?

13. The sum of three consecutive odd integers is 189. What are the integers?

14. The sum of three consecutive odd integers is 255. What are the integers?

15. Find three consecutive odd integers such that the sum of the first, two times the second, and three times the third is 70.

16. The second angle of a triangle is the same size as the first angle. The third angle is 12 degrees larger than the first angle. How large are the angles?

17. Two angles of a triangle are the same size. The third angle is 12 degrees smaller than the first angle. Find the measure the angles.

18. Two angles of a triangle are the same size. The third angle is 3 times as large as the first. How large are the angles?

19. The third angle of a triangle is the same size as the first. The second angle is 4 times the third. Find the measure of the angles.

20. The second angle of a triangle is 3 times as large as the first angle. The third angle is 30 degrees more than the first angle. Find the measure fo the angles.

21. The second angle of a triangle is twice as large as the first. The measure of the third angle is 20 degrees greater than the first. How large are the angles?

22. The second angle of a triangle is three times as large as the first. The measure of the third angle is 40 degrees greater than that of the first angle. How large are the three angles?

23. The second angle of a triangle is five times as large as the first. The measure of the third angle is 12 degrees greater than that of the first angle. How large are the angles?

24. The second angle of a triangle is three times the first, and the third is 12 degrees less than twice the first. Find the measures of the angles.

25. The second angle of a triangle is four times the first and the third is 5 degrees more than twice the first. Find the measures of the angles.

26. The perimeter of a rectangle is 150 cm. The length is 15 cm greater than the width. Find the dimensions.

27. The perimeter of a rectangle is 304 cm. The length is 40 cm longer than the width. Find the length and width.

28. The perimeter of a rectangle is 152 meters. The width is 22 meters less than the length. Find the length and width.

29. The perimeter of a rectangle is 280 meters. The width is 26 meters less than the length. Find the length and width.

30. The perimeter of a college basketball court is 96 meters and the length is 14 meters more than the width. What are the dimensions?

31. A mountain cabin on 1 acre of land costs 30,000 dollars. If the land cost 4 times as much as the cabin, what was the cost of each?

32. A horse and a saddle cost 5000 dollars. If the horse cost 4 times as much as the saddle, what was the cost of each?

33. A bicycle and a bicycle helmet cost 240 dollars. How much did each cost, if the bicycle cost 5 times as much as the helmet?

34. Of 240 stamps that harry and his sister collected, Harry collected 3 times as many as his sisters. How many did each collect?

35. If Mr. Brown and his son together had 220 dollars, and Mr. Brown had 10 times as much as his son, how much money had each?

36. In a room containing 45 students there were twice as many girls as boys. How many of each were there?

37. Aaron had 7 tims as many sheep as Beth, and both together had 608. How many sheep had each?

38. A man bought a cow and a calf for 990 dollars, paying 8 times as much for the cow as for the calf. what was the cost of each?

39. Jamal and Moshe began a business with a capital of 7500 dollars. If Jamal furnished half as much capital as Moshe, how much did each furnish?

40. A lab technician cuts a 12 inch piece off tubing into two pieces in such a way that one piece is 2 times longer than the other.

41. A 6 ft board is cut into two pieces, on twice as long as the other. How long are the pieces?

42. An eight ft board is cut into two pieces. One piece is 2 ft longer thant he other. How long are the pieces?

43. An electrician cuts a 30 ft piece of wire into two pieces. One piece is 2 ft longer than the other. How long are the pieces?

44. The total cost for tuition plus room and board at State University is 2,584 dollars. Tuition costs 704 dollars more than room and board. What is the tuition fee?

45. The cost of a private pilot course is 1,275 dollars. The flight portion costs 625 dollars more than the groung school portion. What is the cost of each?

1.9

Solving Linear Equations - Age Problems

An application of linear equations is what are called age problems. When we are solving age problems we generally will be comparing the age of two people both now and in the future (or past). Using the clues given in the problem we will be working to find their current age. There can be alot of information in these problems and we can easily get lost in all the information. To help us organize and solve our problem we will fill out a three by three table for each problem. An example of the basic structure of the table is below

	Age Now	Change
Person 1		
Person 2		

Table 6. Structure of Age Table

Normally where we see "Person 1" and "Person 2" we will use the name of the person we are talking about. We will use this table to set up the following example.

Example 70.

Adam is 20 years younger than Brian. In Two years Brian will be twice as old as Adam. How old are they now?

	Age Now	$+2$
Adam		
Brian		

We use Adam and Brian for our Persons
We use $+2$ for change because the second phrase is two years in the future

	Age Now	$+2$
Adam	$x-20$	
Brain	x	

Consider the "Now" part, **Adam is 20 years youger than Brian**. We are given information about Adam, not Brian. So Brian is x now. To show Adam is 20 years younger we subtract 20, Adam is $x-20$.

	Age Now	$+2$
Adam	$x-20$	$x-20+2$
Brian	x	$x+2$

Now the $+2$ column is filled in. This is done by adding 2 to both Adam$'s$ and Brian$'s$ now column as shown in the table.

	Age Now	$+2$
Adam	$x-20$	$x-18$
Brian	x	$x+2$

Combine like terms in Adam$'s$ future age: $-20+2$
This table is now filled out and we are ready to try and solve.

$B = 2A$	Our equation comes from the future statement: **Brian will be twice as old as Adam.** This means the younger, Adam, needs to be multiplied by 2.
$(x + 2) = 2(x - 18)$	Replace B and A with the information in their future cells, Adam (A) is replaced with $x - 18$ and Brian (B) is replaced with $(x + 2)$ This is the equation to solve!
$x + 2 = 2x - 36$	Distribute through parenthesis
$\underline{\boldsymbol{-x} \quad \boldsymbol{-x}}$	Subtract x from both sides to get variable on one side
$2 = x - 36$	Need to clear the -36
$\underline{\mathbf{+36} \quad \mathbf{+36}}$	Add 36 to both sides
$38 = x$	Our solution for x

	Age now
Adam	$38 - 20 = 18$
Brian	38

The first column will help us answer the question. Replace the $x's$ with 38 and simplify. Adam is 18 and Brian is 38

Solving age problems can be summarized in the following five steps. These five steps are guidelines to help organize the problem we are trying to solve.

1. Fill in the now collumn. The person we know nothing about is x.

2. Fill in the future/past collumn by adding/subtracting the change to the now collumn.

3. Make an equation for the relationship in the future. This is independent of the table.

4. Replace variables in equation with information in future cells of table

5. Solve the equation for x, use the solution to answer the question

These five steps can be seen illustrated in the following example.

Example 71.

Carmen is 12 years older than David. Five years ago the sum of their ages will be 28. How old are they now?

	Age Now	-5
Carmen		
David		

Five years ago is -5 in the change collumn.

	Age Now	-5
Carmen	$x + 12$	
David	x	

Carmen is 12 years older than David. We don$'t$ know about David so he is x, Carmen then is $x + 12$

	Age Now	-5
Carmen	$x+12$	$x+12-5$
David	x	$x-5$

Subtract 5 from now collumn to get the change

	Age Now	-5
Carmen	$x+12$	$x+7$
David	x	$x-5$

Simplify by combining like terms $12-5$
Our table is ready!

$C+D=28$ The sum of their ages will be 29. So we add C and D

$(x+7)+(x-5)=28$ Replace C and D with the change cells.

$x+7+x-5=28$ Remove parenthesis

$2x+2=28$ Combine like terms $x+x$ and $7-5$

$\underline{\mathbf{-2\ \ -2}}$ Subtract 2 from both sides

$2x=26$ Notice x is multiplied by 2

$\overline{\mathbf{2}}\ \ \overline{\mathbf{2}}$ Divide both sides by 2

$x=13$ Our solution for x

	Age Now
Caremen	$13+12=25$
David	13

Replace x with 13 to answer the question
Caremen is 25 and David is 13

Sometimes we are given the sum of their ages right now. These problems can be tricky. In this case we will right the sum above the now collumn and make the first person's age now x. The second person will then turn into the subtraction problem total $-x$. This is shown in Example 3

Example 72.

The sum of the ages of Nicole and Kristen is 32. In two years Nicole will be three times as old as Kristin. How old are they now?

32

	Age Now	$+2$
Nicole	x	
Kristen	$32-x$	

The change is $+2$ for two years in the future
The total is placed above Age Now
The first person is x. The second becomes $32-x$

	Age Now	$+2$
Nicole	x	$x+2$
Kristen	$32-x$	$32-x+2$

Add 2 to each cell fill in the change collumn

	Age Now	$+2$
Nicole	x	$x+2$
Kristen	$32-x$	$34-x$

Combine like terms $32+2$, our table is done!

$N=3K$ Nicole is three times as old as Kristen.

$$(x+2)=3(34-x)$$ Replace variables with information in change cells
$$x+2=102-3x$$ Distribute through parenthesis
$$\underline{+\mathbf{3x} \qquad +\mathbf{3x}}$$ Add $3x$ to both sides so variable is only on one side
$$4x+2=102$$ Solve the two − step equation
$$\underline{-\mathbf{2} \quad -\mathbf{2}}$$ Subtract 2 from both sides
$$4x=100$$ The variable is multiplied by 4
$$\overline{\mathbf{4}} \quad \overline{\mathbf{4}}$$ Divide both sides by 4
$$x=25$$ Our solution for x

	Age Now
Nicole	25
Kristen	$32-25=7$

Plug 25 in for x in the now collumn
Nicole is 25 and Kristen is 7

A slight variation on age problems is to ask not how old the people are, but rather ask how long until we have some relationship about their ages. In this case we alter our table slightly. In the change collumn because we don't know the time to add or subtract we will use a variable, t, and add or subtract this from the now collumn. This is shown in Example 4.

Example 73.

Louis is 26 years old. Her daughter is 4 years old. In how many years will Louis be double her daughter's age?

	Age Now	$+t$
Louis	26	
Daughter	4	

As we are given their ages now, these numbers go into the table. The change is unknown, so we write $+t$ for the change

	Age Now	$+t$
Louis	26	$26+t$
Daughter	4	$4+t$

Fill in the change collumn by adding t to each person$'s$ age. Our table is now complete.

$$L=2D$$ Lois will be double her daughter
$$(26+t)=2(4+t)$$ Replace variables with information in change cells
$$26+t=8+2t$$ Distribute through parentesis
$$\underline{-\boldsymbol{t} \qquad -\boldsymbol{t}}$$ Subtract t from both sides
$$26=8+t$$ Now we have an 8 added to the t
$$\underline{-\mathbf{8}-\mathbf{8}}$$ Subtract 8 from both sides
$$18=t$$ In 18 years she will be double her daughter$'s$ age

Age problems have several steps to them. However, if we take the time to work through each of the steps carefully, keeping the information organized, the problems can be solved quite nicely.

Practice - Age Problems

1. A boy is 10 years older than his brother. In 4 years he will be twice as old as his brother. Find the present age of each.

2. A father is 4 times as old as his son. In 20 years the father will be twice as old as his son. Find the present age of each.

3. Pat is 20 years older than his son James. In two years Pat will be twice as old as James. How old are they now?

4. Diane is 23 years older than her daughter Amy. In 6 years Diane will be twice as old as Amy. How old are they now?

5. Fred is 4 years older than Barney. Five years ago the sum of their ages was 48. How old are they now?

6. John is four times as old as Martha. Five years ago the sum of their ages was 50. How old are they now?

7. Tim is 5 years older than JoAnn. Six years from now the sumof their ages will be 79. How old are they now?

8. Jack is twice as old as Lacy. In three years the sum of their ages will be 54. How old are they now?

9. The sum of the ages of John and Mary is 32. Four years ago, John was twice as old as Mary. Find the present age of each.

10. The sum of the ages of a father and son is 56. Four years ago the father was 3 times as old as the son. Find the present age of each.

11. The sum of the ages of a china plate and a glass plate is 16 years. Four years ago the china plate was three times the age of the glass plate. Find the present age of each plate.

12. The sum of the ages of a wood plaque and a bronze plaque is 20 years. Four

years ago, the bronze plaque was one-half the age of the wood plaque. Find the present age of each plaque.

13. A is now 34 years old, and B is 4 years old. In how many years will A be twice as old as B?

14. A man's age is 36 and that of his daughter is 3 years. In how many years will the man be 4 times as old as his daughter?

15. An Oriental rug is 52 years old and a Persian rug is 16 years old. How many years ago was the Oriental rug four times as old as the Persian Rug?

16. A log cabin quilt is 24 years old and a friendship quilt is 6 years old. In how may years will the log cabin quilt be three times as old as the friendship quilt?

17. The age of the older of two boys is twice that of the younger; 5 years ago it was three times that of the younger. Find the age of each.

18. A pitcher is 30 years old, and a vase is 22 years old. How many years ago was the pitcher twice as old as the vase?

19. Marge is twice as old as Consuelo. The sum of their ages seven years ago was 13. How old are they now?

20. The sum of Jason and Mandy's age is 35. Ten years ago Jason was double Mandy's age. How old are they now?

21. A silver coin is 28 years older tha a bronze coin. In 6 years, the silver coin will be twice as old as the bronze coin. Find the present age of each coin.

22. A sofa is 12 years old and a table is 36 years old. In how many years will the sofa be twice as old as the table?

23. A limestone statue is 56 years older than a marble statue. In 12 years, the limestone will be three times as old as the marble statue. Find the present age of the statue.

24. A pewter bowl is 8 years old, and a silver bowl is 22 years old. In how many years will the silver bowl be twice the age of the pewter bowl?

25. Brandon is 9 years older than Ronda. In four years the sum of their ages will be 91. How old are they now?

26. A kerosene lamp is 95 years old, and an electric lamp is 55 years old. How many years ago was the kerosene lamp twice the age of the electric lamp?

27. A father is three tims as old as his son, and his daughter is 3 years younger

than the son. If the sum of their ages 3 years ago was 63 years, find the present age of the father.

28. The sum of Clyde and Wendy's age is 55. In four years, Wendy will be three times as old as Clyde. How old are they now?

29. The sum of the ages of two ships is 12 years. Two years ago, the age of the older ship was three times the age of the newer ship. Find the present age of each ship.

30. Chelsea's age is double Daniel's age. Eight years ago the sum of their ages was 32. How old are they now?

31. Ann is eighteen years older than her son. One year ago, she was three tims as old as her son. How old are they now?

32. The sum of the ages of Kristen and Ben is 32. Four years ago Kristen was twice as old as Ben. How old are they both now?

33. A mosaic is 74 years older than the engraving. Thirty years ago, the mosaic was three times as old as the engraving. Find the present age of each.

34. The sum of the ages of Elli and Dan is 56. Four years ago Elli was 3 times as old as Dan. How old are they now?

35. A wool tapestry is 32 years older than a linen tapestry. Twenty years ago, the wool tapestry was twice as old as the linen tapestry. Find the present age of each.

36. Carolyn's age is triple her daughter's age. In eight years the sum of their ages will be 72. How old are they now?

37. Nicole is 26 years old. Emma is 2 years old. In how many years will Nicole be triple Emma's age?

38. The sum of the ages of two children is 16 years. Four years ago, the age of the older child was three times the age of the younger child. Find the present age of each child.

39. Mike is 4 years older than Ron. In two years, the sum of their ages will be 84. How old are they now?

40. A marble bust is 25 years old, and a terra-cotta bust is 85 years old. In how many years will the terra-cotta bust be three times as old as the marble bust?

1.10

Solving Linear Equations - Distance

An application of linear equations can be found in distance problems. When solving distance problems we will use the relationship $rt = d$ or rate (speed) times time equals distance. For example, if a person were to travel 30 mph for 4 hours. To find the total distance we would multiply rate times time or $(30)(4) = 120$. This person travel a distance of 120 miles. The problems we will be solving here will be a few more steps than described above. So to keep the information in the problem organized we will use a table. An example of the basic structure of the table is blow:

	Rate	Time	Distance
Person 1			
Person 2			

Table 7. Structure of Distance Problem

The third collumn, distance, will always be filled in by multiplying the rate and time collumns together. If we are given a total distance of both persons or trips we will put this information below the distance collumn. We will now use this table to set up and solve the following example

Example 74.

Two joggers start from opposite ends of an 8 mile course running towards each other. One jogger is running at a rate of 4 mph, and the other is running at a rate of 6 mph. After how long will the joggers meet?

	Rate	Time	Distance
Jogger 1			
Jogger 2			

The basic table for the joggers, one and two

	Rate	Time	Distance
Jogger 1	**4**		
Jogger 2	**6**		

We are given the rates for each jogger.
These are added to the table

	Rate	Time	Distance
Jogger 1	4	$\boldsymbol{t}$	
Jogger 2	6	$\boldsymbol{t}$	

We only know they both start and end at the Same time. We use the variable t for both times

	Rate	Time	Distance
Jogger 1	4	t	$\boldsymbol{4t}$
Jogger 2	6	t	$\boldsymbol{6t}$
			8

The distance collumn is filled in by multiplying rate by time

We have **total distance**, 8 miles, under distance

$4t + 6t = 8$ The distance collumn gives equation by adding

$10t = 8$ Combine like terms, $4t + 6t$

$\overline{\mathbf{10}}\ \overline{\mathbf{10}}$ Divide both sides by 10

$t = \frac{4}{5}$ Our solution for t, $\frac{4}{5}$ hour (48 minutes)

As the example illustrates, once the table is filled in, the equation to solve is very easy to find. This same process can be seen in the following example

Example 75.

Bob and Fred start from the same point and walk in opposite directions. Bob walks 2 miles per hour faster than Fred. After 3 hours they are 30 miles apart. How fast did each walk?

	Rate	Time	Distance
Bob		**3**	
Fred		**3**	

The basic table with given times filled in
Both traveled 3 horus

	Rate	Time	Distance
Bob	$\boldsymbol{r+2}$	3	
Fred	$\boldsymbol{r}$	3	

Bob walks 2 mph faster than Fred
We know nothing about Fred, so use r for his rate
Bob is $r+2$, showing 2 mph faster

	Rate	Time	Distance
Bob	$r+2$	3	$\boldsymbol{3r+6}$
Fred	r	3	$\boldsymbol{3r}$

Distance column is filled in by multiplying rate by Time. Be sure to distribute the $3(r+2)$ for Bob.

30 — **Total distance** is put under distance

$3r+6+3r=30$ — The distance columns is our equation, by adding

$6r+6=30$ — Combine like terms $3r+3r$

$\underline{\mathbf{-6\quad -6}}$ — Subtract 6 from both sides

$6r=24$ — The variable is multiplied by 6

$\overline{\mathbf{6}}\quad\overline{\mathbf{6}}$ — Divide both sides by 6

$r=4$ — Our solution for r

	Rate
Bob	$4+2=6$
Fred	4

To answer the question completely we plug 4 in for r in the table. Bob traveled 6 miles per hour and Fred traveled 4 mph

Some problems will require us to do a bit of work before we can just fill in the cells. One exampmle of this is if we are given a total time, rather than the individual times like we had in the previous example. If we are given total time will we write this above the time collumn, use t for the first person's time, and make a subtraction problem, Total $-\,t$, for the second person's time. This is shown in the next example

Example 76.

Two campers left their campsite by conoe and paddled downstream at an average speed of 12 mph. The turned around and paddled back upstream at an average rate of 4 mph. The total trip took 1 hour. After how much time did the campers turn around downstream?

	Rate	Time	Distance
Down	**12**		
Up	**4**		

Basic table for down and upstream
Given rates are filled in

1

	Rate	Time	Distance
Down	12	$\boldsymbol{t}$	
Up	4	$\mathbf{1}-\boldsymbol{t}$	

Total time is put above time collumn
As we have the total time, in the first time we have t, the second time becomes the subtraction, total $- t$

	Rate	Time	Distance
Down	12	t	$\mathbf{12}\boldsymbol{t}$ =
Up	4	$1-t$	$\mathbf{4}-\mathbf{4}\boldsymbol{t}$

Distance collumn is found by multiplying rate by time. Be sure to distribute $4(1-t)$ for upstream. As they cover the **same distance**, $=$ is put after the down distance

$$12t = 4 - 4t$$ With equals sign, distance collum is equation

$$\underline{+\mathbf{4}\boldsymbol{t} \quad +\mathbf{4}\boldsymbol{t}}$$ Add $4t$ to both sides so varaible is only on one side

$$16t = 4$$ Variable is multiplied by 16

$$\overline{\mathbf{16}} \quad \overline{\mathbf{16}}$$ Divide both sides by 16

$$t = \frac{1}{4}$$ Our solution, turn around after $\frac{1}{4}$ hr (15 min)

Another type of distance problem where we do some work is when one person catches up with another. Here a slower person has a head start and the faster person is trying to catch up with him or her and we want to know how long it will take the fast person to do this. Our startegy for this problem will be to use t for the faster person's time, and add amount of time the head start was to get the slower person's time. This is shown in the next example.

Example 77.

Mike leaves his house traveling 2 miles per hour. Joy leaves 6 hours later to catch up with him traveling 8 miles per hour. How long will it take her to catch up with him?

	Rate	Time	Distance
Mike	**2**		
Joy	**8**		

Basic table for Mike and Joy
The given rates are filled in

	Rate	Time	Distance
Mike	2	$\boldsymbol{t}+\mathbf{6}$	
Joy	8	$\boldsymbol{t}$	

Joy, the faster person, we use t for time
Mike$'s$ time is $t + 6$ showing his 6 hour head start

	Rate	Time	Distance
Mike	2	$t+6$	$\mathbf{2}\boldsymbol{t}+\mathbf{12}$ =
Joy	8	t	$\mathbf{8}\boldsymbol{t}$

Distance column is found by multiplying the rate by time. Be sure to distribute the $2(t+6)$ for Mike
As they cover the **same distance**, $=$ is put after Mike$'s$ distance

$2t+12=8t$	Now the distance column is the equation
$\underline{-\mathbf{2t}\quad -\mathbf{2t}}$	Subtract $2t$ from both sides
$12=6t$	The variable is multiplied by 6
$\overline{\mathbf{6}}\ \overline{\mathbf{6}}$	Divide both sides by 6
$2=t$	Our solution for t, she catches him after 2 hours

As these example have shown, using the table can help keep all the given information organized, help fill in the cells, and help find the equation we will solve. The final example clearly illustrates this.

Example 78.

On a 130 mile trip a car travled at an average speed of 55 mph and then reduced its speed to 40 mph for the remainder of the trip. The trip took 2.5 hours. For how long did the car travel 40 mph?

	Rate	Time	Distance
Fast	**55**		
Slow	**40**		

Basic table for fast and slow speeds
The given rates are filled in

2.5

	Rate	Time	Distance
Fast	55	$\boldsymbol{t}$	
Slow	40	$\mathbf{2.5}-\boldsymbol{t}$	

Total time is put above the time collumn
As we have total time, the first time we have t
The second time is the subtraction problem
$2.5-t$

2.5

	Rate	Time	Distance
Fast	55	t	$\mathbf{55}\boldsymbol{t}$
Slow	40	$2.5-t$	$\mathbf{100}-\mathbf{40}\boldsymbol{t}$
			130

Distance collumn is found by multiplying rate by tim. Be sure to distribute $40(2.5-t)$ for slow

Total distance is put under distance

$55t+100-40t=130$	The distance collumn gives our equation by adding
$15t+100=130$	Combine like terms $55t-40t$
$\underline{-\mathbf{100}-\mathbf{100}}$	Subtract 100 from both sides
$15t=30$	The variable is multiplied by 30
$\overline{\mathbf{15}}\ \overline{\mathbf{15}}$	Divide both sides by 15
$t=2$	Our solution for t.

	Time
Fast	2
Slow	$2.5-2=0.5$

To answer the question we plug 2 in for t
The car traveled 40 mph for 0.5 hours (30 minutes)

Practice - Distance, Rate, and Time Problems

1. A is 60 miles from B. An automobile at A starts for B at the rate of 20 miles an hour at the same time that an automobile at B starts for A at the rate of 25 miles an hour. How long will it be before the automobiles meet?

2. Two automobiles are 276 miles apart and start at the same time to travel toward each other. They travel at rates differing by 5 miles per hour. If they meet after 6 hours, find the rate of each.

3. Two trains travel toward each other from points which are 195 miles apart. They travel at rate of 25 and 40 miles an hour respectively. If they start at the same time, how soon will they meet?

4. A and B start toward each other at the same time from points 150 miles apart. If A went at the rate of 20 miles an hour, at what rate must B travel if they meet in 5 hours?

5. A passenger and a freight train start toward each other at the same time from two points 300 miles apart. If the rate of the passenger train exceeds the rate of the freight train by 15 miles per hour, and they meet after 4 hours, what must the rate of each be?

6. Two automobiles started at the same time from point, but traveled in opposite directions. Their rates were 25 and 35 miles per hour respectively. After how many hours were they 180 miles apart?

7. A man having ten hours at his disposal made an excursion, riding out at the rate of 10 miles an hour and returning on foot, at the rate of 3 miles an hour.

Find the distance he rode.

8. A man walks at the rate of 4 miles per hour. How far can he walk into the country and ride back on a trolley that travels at the rate of 20 miles per hour, if he must be back home 3 hours from the time he started?

9. A boy rides away from home in an automobile at the rate of 28 miles an hour and walks back at the rate of 4 miles an hour. The round trip requires 2 hours. How far does he ride?

10. A motorboat leaves a harbor and travels at an average speed of 15 mph toward an island. The average speed on the return trip was 10 mph. How far was the island from the harbor if the total trip took 5 hours?

11. A family frove to a resort at an average speed of 30 mph and later returned over the same road at an average speed of 50 mph. Find the distance to the resort if the total driving time was 8 hours.

12. As part of his flight trainging, a student pilot was required to fly to an airport and then return. The average speed to the airport was 90 mph, and the average speed returning was 120 mph. Find te distance between the two airports if the total flying time was 7 hours.

13. A, who travels 4 miles an hour starts from a certain place 2 hours in advance of B, who travels 5 miles an hour in the same direction. How many hours must B travel to overtake A?

14. A man travels 5 miles an hour. After traveling for 6 hours another man starts at the same place, following at the rate of 8 miles an hour. When will the second man overtake the first?

15. A motorboat leaves a harbor and travels at an average speed of 8 mph toward a small island. Two hours later a cabin cruiser leaves the same harbor and travels at an average speed of 16 mph toward the same island. In how many hours after the cabin cruiser leaves will the cabin cuiser be alongside the motorboat?

16. A long distance runner started on a course running at an average speed of 6 mph. One hour later, a second runner began the same course at an average speed of 8 mph. How long after the second runner started will the second runner overtake the first runner?

17. A car traveling at 48 mph overtakes a cyclist who, riding at 12 mph, has had a 3 hour head start. How far from the starting point does the car overtake the cyclist?

18. A jet plane traveling at 600 mph overtakes a propeller-driven plane which has

had a 2 hour head start. The propeller-driven plane is traveling at 200 mph. How far from the starting point does the jet overtake the propeller-driven plane?

19. Two men are traveling in opposite directions at the rate of 20 and 30 miles an hour at the same time and from the same place. In how many hours will they be 300 miles apart?

20. Running at an average rate of 8 m/s, a sprinter ran to the end of a track and then jogged back to the starting point at an average rate of 3 m/s. The sprinter took 55 s to run to the end of the track and jog back. Find the length of the track.

21. A motorboat leaves a harbor and travels at an average speed of 18 mph to an island. The average speed on the return trip was 12 mph. How far was the island from the harbor if the total trip took 5 h?

22. A motorboat leaves a harbor and travels at an average speed of 9 mph toward a small island. Two hours later a cabin cruiser leaves the same harbor and travels at an average speed of 18 mph toward the same island. In how many hours after the cabin cruiser leaves will the cabin cruiser be alongside the motorboat?

23. A jet plane traveling at 570 mph overtakes a propeller-driven plane that has had a 2 h head start. The propeller-driven plane is traveling at 190 mph. How far from the starting point does the jet overtake the propeller-driven plane?

24. Two trains start at the same time from the same place and travel in opposite directions. If the rate of one is 6 miles per hour more than the rate of the other and they are 168 miles apart at the end of 4 horus, what is the rate of each?

25. As part of flight traning, a student pilot was required to fly to an airport and then return. The average speed on the way to the airport was 100 mph, and the average speed returning was 150 mph. Find the distance between the two airports if the total flight time was 5 h.

26. Two cyclists start from the same point and ride in opposite directions. One cyclist rides twice as fast as the other. In three hours they are 72 miles apart. Find the rate of each cyclist.

27. A car traveling at 56 mph overtakes a cyclist who, riding at 14 mph, has had a 3 h head start. How far from the starting point does the car overtake the cyclist?

28. Two small planes start from the same point and fly in opposite directions.

The first plan is flying 25 mph slower than the second plane. In two hours the planes are 430 miles apart. Find the rate of each plane.

29. A bus traveling at a rate of 60 mph overtakes a car traveling at a rate of 45 mph. If the car had a 1 h head start, how far from the starting point does the bus overtake the car?

30. Two small planes start from the same point and fly in opposite directions. The first plane is flying 25 mph slower than the second plane. In 2 h, the planes are 470 mi apart. Find the rate of each plane.

31. A truck leaves a depot at 11 A.M. and travels at a speed of 45 mph. At noon, a van leaves the same place and travels the same route at a speed of 65 mph. At what time does the van overtake the truck?

32. A family drove to a resort at an average speed of 25 mph and later returned over the same road at an average speed of 40 mph. Find the distance to the resort if the total driving time was 13 h.

33. Three campers left their campsite by canoe and paddled downstream at an average rate of 10 mph. They then turned around and paddled back upstream at an average rate of 5 mph to return to their campsite. How long did it take the campers to canoe downstream if the total trip took 1 hr?

34. A motorcycle breaks down and the rider has to walk the rest of the way to work. The motorcycle was being driven at 45 mph, and the rider walks at a speed of 6 mph. The distance from home to work is 25 miles, and the total time for the trip was 2 hours. How far did the motorcycle go before if broke down?

35. A student walks and jogs to college each day. The student averages 5 km/hr walking and 9 km/hr jogging. The distance from home to college is 8 km, and the student makes the trip in one hour. How far does the student jog?

36. On a 130 mi trip, a car traveled at an average speed of 55 mph and then reduced its speed to 40 mph for the remainder of the trip. The trip took a total of 2.5 h. For how long did the car travel at 40 mph?

37. On a 220 mi trip, a car traveled at an average speed of 50 mph and then reduced its average speed to 35 mph for the remainder fo the trip. The trip took a total of 5 h. How long did the car travel at each speed?

38. An executive drove from home at an average speed of 40 mph to an airport where a helicopter was waiting. The executive boarded the helicopter adn flew to the corporate offices at and average speed of 60 mph. The entire distance was 150 mi. The entire trip took 3 h. Find the distance from the airport to the corporate offices.

Answers to One-Step Equations

1) $\{7\}$
2) $\{11\}$
3) $\{-5\}$
4) $\{4\}$
5) $\{10\}$
6) $\{6\}$
7) $\{-19\}$
8) $\{-6\}$
9) $\{18\}$
10) $\{6\}$
11) $\{-20\}$
12) $\{-7\}$
13) $\{-108\}$
14) $\{5\}$
15) $\{-8\}$
16) $\{4\}$
17) $\{17\}$
18) $\{4\}$
19) $\{20\}$
20) $\{-208\}$
21) $\{3\}$
22) $\{16\}$
23) $\{-13\}$
24) $\{-9\}$
25) $\{15\}$
26) $\{8\}$
27) $\{-10\}$
28) $\{-204\}$
29) $\{5\}$
30) $\{2\}$
31) $\{-11\}$
32) $\{-14\}$
33) $\{14\}$
34) $\{1\}$
35) $\{-11\}$
36) $\{-15\}$
37) $\{-240\}$
38) $\{-135\}$
39) $\{-16\}$
40) $\{-380\}$

Answers to Two-Step Equations

1) $\{-4\}$
2) $\{7\}$
3) $\{-14\}$
4) $\{-2\}$
5) $\{10\}$
6) $\{-12\}$
7) $\{0\}$
8) $\{12\}$
9) $\{-10\}$
10) $\{-16\}$
11) $\{14\}$
12) $\{-7\}$
13) $\{4\}$
14) $\{-5\}$
15) $\{16\}$
16) $\{-15\}$
17) $\{7\}$
18) $\{12\}$
19) $\{9\}$
20) $\{0\}$
21) $\{11\}$
22) $\{-6\}$
23) $\{-10\}$
24) $\{13\}$
25) $\{1\}$
26) $\{4\}$
27) $\{-9\}$
28) $\{15\}$
29) $\{-6\}$
30) $\{6\}$
31) $\{-16\}$
32) $\{-4\}$
33) $\{8\}$
34) $\{-13\}$
35) $\{-2\}$
36) $\{10\}$
37) $\{-12\}$
38) $\{0\}$
39) $\{12\}$
40) $\{-9\}$

Answers to General Linear Equations

1) $\{-3\}$
2) $\{6\}$
3) $\{7\}$
4) $\{0\}$
5) $\{1\}$
6) $\{3\}$
7) $\{5\}$
8) $\{-4\}$
9) $\{0\}$
10) $\{3\}$
11) $\{1\}$
12) {All real numbers}
13) $\{8\}$
14) $\{1\}$
15) $\{-7\}$
16) $\{0\}$
17) $\{2\}$
18) $\{-3\}$
19) $\{-3\}$
20) $\{3\}$
21) $\{3\}$
22) $\{-1\}$
23) $\{-3\}$
24) $\{-1\}$
25) $\{8\}$
26) $\{0\}$
27) $\{-1\}$
28) $\{5\}$
29) $\{-1\}$
30) $\{1\}$
31) $\{-4\}$
32) $\{0\}$
33) $\{-3\}$
34) $\{0\}$
35) $\{0\}$
36) $\{-2\}$
37) $\{-6\}$
38) $\{-3\}$
39) $\{5\}$
40) $\{6\}$
41) $\{0\}$
42) $\{-2\}$
43) {No Solution}
44) $\{0\}$
45) $\{12\}$
46) {All real numbers}
47) {No Solution}
48) $\{1\}$
49) $\{-9\}$
50) $\{0\}$

Answers to Fractions

1) $\{\frac{3}{4}\}$
2) $\{-\frac{4}{3}\}$
3) $\{\frac{6}{5}\}$
4) $\{\frac{1}{6}\}$
5) $\{-\frac{19}{6}\}$
6) $\{\frac{25}{8}\}$
7) $\{-\frac{7}{9}\}$
8) $\{-\frac{1}{3}\}$
9) $\{-2\}$
10) $\{\frac{3}{2}\}$
11) $\{0\}$
12) $\{\frac{4}{3}\}$
13) $\{-\frac{3}{2}\}$
14) $\{\frac{1}{2}\}$
15) $\{-\frac{4}{3}\}$
16) $\{1\}$
17) $\{0\}$
18) $\{-\frac{5}{3}\}$
19) $\{1\}$
20) $\{1\}$
21) $\{\frac{1}{2}\}$
22) $\{-1\}$
23) $\{-2\}$
24) $\{-\frac{9}{4}\}$
25) $\{-\frac{7}{2}\}$
26) $\{-\frac{1}{2}\}$

27) $\{-\frac{5}{3}\}$

28) $\{-\frac{3}{2}\}$

29) $\{\frac{4}{3}\}$

30) $\{\frac{3}{2}\}$

Answers - Formulas

1. $b=\frac{c}{a}$
2. h = gi
3. $x=\frac{\text{gb}}{f}$
4. $y=\frac{\text{pq}}{3}$
5. $x=\frac{a}{3b}$
6. $y=\frac{\text{cb}}{2m}$
7. $m=\frac{E}{c^2}$
8. $D=\frac{\text{ds}}{S}$
9. $\pi=\frac{3V}{4r^3}$
10. $m=\frac{2E}{v_2}$
11. $c=b-a$
12. x = g + f
13. $y=\frac{\text{cm}+\text{cn}}{4}$
14. $r=\frac{k(a-3)}{5}$
15. $D=\frac{12V}{\pi n}$
16. $k=\frac{F}{R-L}$
17. $n=\frac{P}{p-c}$
18. $L=S-2B$
19. $D=\text{TL}+d$
20. $E_a=\text{IR}+\text{Eg}$
21. $L_o=\frac{L}{1+\text{at}}$
22. $x=\frac{c-b}{a}$
23. $m=\frac{p-q}{2}$
24. $L=\frac{q+6p}{6}$
25. k = qr + m
26. $T=\frac{R-b}{a}$
27. $v=\frac{16t^2+h}{t}$
28. $h=\frac{s-\pi r^2}{\pi r}$
29. $Q_2=\frac{Q_1+\text{PQ}_1}{P}$
30. $r_1=\frac{L-2d-\pi r^2}{\pi}$
31. $T_1=\frac{\text{Rd}-\text{kAT}_2}{\text{kA}}$
32. $v_2=\frac{\text{Pg}+V_1^2}{V_1}$
33. $a=\frac{c-b}{x}$
34. $r=\frac{d}{t}$
35. $m=\frac{V}{\text{Ih}}$
36. $h=\frac{3v}{\pi r^2}$
37. $a=\frac{c-1}{b}$
38. $b=\frac{c-1}{a}$
39. $b=\frac{b2+s}{a}$
40. $w=\frac{\text{at}-s}{b}$
41. $x=\frac{c}{a+b}$
42. $x=3-5y$
43. $y=\frac{3-x}{5}$
44. $x=\frac{7-2y}{3}$
45. $y=\frac{7-3x}{2}$
46. $a=\frac{7b+4}{5}$
47. $b=\frac{5a-4}{7}$
48. $x=\frac{8+5y}{4}$
49. $y=\frac{4x-8}{5}$
50. $f=\frac{9c+160}{5}$

Answers to Absolute Value Equations

1) No solution. $\varnothing$

2) No solution. $\varnothing$

3) $\{4,-4\}$

4) $\{6,\ -6\}$

5) $\{7,\ -7\}$

6) $\{6,\ -6\}$

7) $\{35,\ -35\}$

8) No solution. $\varnothing$

9) No solution. $\varnothing$

10) $\{10,\ -10\}$

11) $\{5,\ -5\}$

12) $\{8,\ -8\}$

13) No solution. $\varnothing$

14) $\{5,\ -5\}$

15) $\{1,\ -1\}$

16) $\{0,\ -8\}$

17) $\{10,\ -10\}$

18) $\{5, 1\}$

19) $\{-8, -12\}$

20) $\{-3, 21\}$

21) $\{4, \ -14\}$

22) $\{\frac{2}{3}, -\frac{2}{3}\}$

23) $\{1, \ -1\}$

24) $\{8, -8\}$

25) $\{2, \ -2\}$

26) $\{7, \ -7\}$

27) $\{8, 6\}$

28) $\{10, -26\}$

29) No solution. $\varnothing$

30) No solution. $\varnothing$

31) $\{-\frac{39}{5}, 5\}$

32) $\{-\frac{19}{3}, 3\}$

33) $\{9, -\frac{15}{2}\}$

34) $\{-4, 6\}$

35) $\{2, -9\}$

36) $\{-\frac{19}{3}, 3\}$

37) $\{-\frac{1}{3}, -3\}$

38) $\{6, -\frac{20}{3}\}$

39) No solution. $\varnothing$

40) $\{7, -17\}$

41) No solution. $\varnothing$

42) $\{\frac{23}{5}, -7\}$

43) No solution. $\varnothing$

44) $\{1, 0\}$

45) $\{-8, \frac{36}{5}\}$

46) $\{-\frac{4}{3}, -\frac{2}{7}\}$

47) $\{-6, \frac{2}{5}\}$

48) $\{7, \ \frac{1}{5}\}$

49) $\{-\frac{22}{5}, -\frac{2}{13}\}$

50) $\{-\frac{19}{22}, -\frac{11}{38}\}$

Answers - Variation

1) $\frac{c}{a} = k$

2) $\frac{x}{\text{yz}} = k$

3) $\text{wx} = k$

4) $\frac{r}{s^2} = k$

5) $\frac{f}{\text{xy}} = k$

6) $\text{jm}^3 = k$

7) $\frac{h}{m} = k$

8) $\frac{x}{a^2\sqrt{b}} = k$

9) $\text{ab} = k$

10) $\frac{a}{b} = 3$

11) $\frac{P}{\text{rq}} = 0.5$

12) $\text{cd} = 28$

13) $\frac{t}{u^2} = 0.67$

14) $\frac{e}{\text{fg}} = 4$

15) $\text{wx}^3 = 1458$

16) $\frac{h}{j} = 1.5$

17) $\frac{a}{x^2\sqrt{y}} = 0.33$

18) mn = 3.78

19) 6 k

20) 5.3 k

21) 33.3 cm

22) 160 kg/cm^3

23) 241,920,000 cans

24) 3.5 hours

25) 4.29 dollars

26) 450 m

27) 40 kg

28) 5.7 hr

29) 40 lb

30) 100 N

31) 27 min

32) 56.2 mph

33) r = 36

34) 66 mph

35) 7.5 m

36) V = 100.5 cm^3

37) 6.25 km

38) I = 0.25

39) 1600 km

Answer Set - Word Problems

1) 11

2) 5

3) -4

4) 32

5) -13
6) 62
7) 16
8) $\frac{17}{4}$
9) 35, 36, 37
10) $-43, -42, -41$
11) $-14, -13, -12$
12) 52, 54
13) 61, 63, 65
14) 83, 85, 87
15) 9, 11, 13
16) 56, 56, 68
17) 64, 64, 52
18) 36, 36, 108
19) 30, 120, 30
20) 30, 90, 60
21) 40, 80, 60
22) 28, 84, 68
23) 24, 120, 36
24) 32, 96, 52
25) 25, 100, 55
26) 45, 30
27) 96, 56
28) 27, 49
29) 57, 83
30) 17, 31
31) 6000, 24000
32) 1000, 4000
33) 40, 200
34) 60, 180
35) 20, 200
36) 30, 15
37) 76, 532
38) 110, 880
39) 2500, 5000
40) 4, 8
41) 2, 4
42) 3, 5
43) 14, 16
44) 1644
45) 325, 950

Answers - Age Problems

1) 6, 16
2) 10, 40
3) 18, 38
4) 17, 40
5) 27, 31
6) 12, 48
7) 31, 36
8) 16, 32
9) 12, 20
10) 40, 16
11) 16, 6
12) 12, 8
13) 26
14) 8
15) 4
16) 3
17) 10, 20
18) 14
19) 9, 18
20) 15, 20
21) 50, 22
22) 12
23) 72, 16
24) 6
25) 37, 46
26) 15
27) 45
28) 12, 52
29) 8, 4
30) 16, 32
31) 10, 28
32) 12,20
33) 141, 67
34) 16, 40
35) 84, 52
36) 14, 42
37) 10
38) 10, 6
39) 39, 42
40) 5

Answers - Distance, Rate, and Time Problems

1) $1\frac{1}{3}$

2) $25\,\frac{1}{2}, 20\,\frac{1}{2}$

3) 3

4) 10

5) 30, 45

6) 3

7) $\frac{300}{13}$

8) 10

9) 7

10) 30

11) 150

12) 360

13) 8

14) 10

15) 2

16) 3

17) 48

18) 600

19) 6

20) 120

21) 36

22) 2

23) 570

24) 24, 18

25) 300

26) 8, 6

27) 56

28) 95, 120

29) 180

30) 105, 130

31) 2:15 PM

32) 200

33) $\frac{1}{3}$

34) 15

35) $\frac{27}{4}$

36) $\frac{1}{2}$

37) 3, 2

38) 90