Beginning and Intermediate Algebra

Chapter 0: Arithmetic

by Tyler Wallace

Chapter 0: Arithmetic Review

0.1

Arithmetic - Integers

The ability to work comfortably with negative numbers is essential to success in algebra. For this reason we will do a quick review of adding, subtracting, multiplying and dividing of integers. **Integers** are all the positive whole numbers, zero, and their opposites (negatives). As this is intended to be a review of integers, the descriptions and examples will not be as detailed as a normal lesson.

When adding integers we have two cases to consider. The first is if the signs match, both positive or both negative. If the signs match we will add the numbers together and keep the sign. This is illustarted in the following examples

Example 1.

$$\begin{array}{rl} -5+(-3) & \text{Same sign, add } 5+3, \text{keep the negative} \\ -8 & \text{Our Solution} \end{array}$$

Example 2.

$$\begin{array}{rl} -7+(-5) & \text{Same sign, add } 7+5, \text{keep the negative} \\ -12 & \text{Our Solution} \end{array}$$

If the signs don't match, one positive and one negative number, we will subtract the numbers (as if they were all positive) and then use the sign from the larger number. This means if the larger number is positive, the answer is positive. If the larger number is negative, the answer is negative. This is shown in the following examples

Example 3.

$$\begin{array}{rl} -7+2 & \text{Different signs, subtract } 7-2, \text{use sign from bigger number, negative} \\ -5 & \text{Our Solution} \end{array}$$

Example 4.

$$\begin{array}{rl} -4+6 & \text{Different signs, subtract } 9-4, \text{use sign from bigger number, positive} \\ 2 & \text{Our Solution} \end{array}$$

Example 5.

$$\begin{array}{rl} 4+(-3) & \text{Different signs, subtract } 4-3\text{, use sign from bigger number, positive} \\ 1 & \text{Our Solution} \end{array}$$

Example 6.

$$\begin{array}{rl} 7+(-10) & \text{Different signs, subtract } 10-7\text{, use sign from bigger number, negative} \\ -3 & \text{Our Solution} \end{array}$$

For subtraction of negatives we will change the problem to an addition problem which we can then solve using the above methods. The way we change a subtraction to an addition is to ad the opposite of the number after the subtraction sign. Often this method is refered to as "add the opposite." This is illustrated in the following examples

Example 7.

$$\begin{array}{rl} 8-3 & \text{Add the opposite of } 3 \\ 8+(-3) & \text{Different signs, subtract } 8-3\text{, use sign from bigger number, positive} \\ 5 & \text{Our Solution} \end{array}$$

Example 8.

$$\begin{array}{rl} -4-6 & \text{Add the opposite of } 6 \\ -4+(-6) & \text{Same sign, add } 4+6\text{, keep the negative} \\ -10 & \text{Our Solution} \end{array}$$

Example 9.

$$\begin{array}{rl} 9-(-4) & \text{Add the opposite of } -4 \\ 9+4 & \text{Same sign, add } 9+4\text{, keep the positive} \\ 13 & \text{Our Solution} \end{array}$$

Example 10.

$$\begin{array}{rl} -6-(-2) & \text{Add the opposite of } -2 \\ -6+2 & \text{Different sign, subtract } 6-2\text{, use sign from bigger number, negative} \\ -4 & \text{Our Solution} \end{array}$$

Multiplication and division of integers both work in a very similar pattern. The short description of the process is we multiply and divide like normal, if the signs match (both positive or both negative) the answer is positive. If the signs don't match (one positive and one negative) then the answer is negative. This is shown in the following examples

Example 11.

$$\begin{array}{rl} (4)(-6) & \text{Signs do not match, answer is negative} \\ -24 & \text{Our Solution} \end{array}$$

Example 12.

$$\begin{array}{rl} \dfrac{-36}{-9} & \text{Signs match, answer is positive} \\ 4 & \text{Our Solution} \end{array}$$

Example 13.

$$\begin{array}{rl} -2(-6) & \text{Signs match, answer is positive} \\ 12 & \text{Our Solution} \end{array}$$

Example 14.

$$\begin{array}{rl} \dfrac{15}{-3} & \text{Signs do not match, answer is negative} \\ -5 & \text{Our Solution} \end{array}$$

A few things to be careful of when working with integers. First be sure not to confuse a problem like $-3-8$ with $-3(-8)$. The second problem is a multiplication problem because there is nothing between the 3 and the parenthesis. If there is no operation written in between the parts, then we assume that means we are multiplying. The $-3-8$ problem, is subtraction because th subtraction separates the 3 from what comes after it. Another item to watch out for is to be careful not to mix up the pattern for adding and subtracting integers with the pattern for multiplying and dividing integers. They can look very similar, for example if the signs match on addition, the we keep the negative, $-3+(-7)=-10$, but if the signs match on multiplication, the answer is positive, $(-3)(-7)=21$.

Practice - Integers

Evaluate each expression.

1) $1-3$

2) $4-(-1)$

3) $(-6)-(-8)$

4) $(-6)+8$

5) $(-3)-3$

6) $(-8)-(-3)$

7) $3-(-5)$

8) $7-7$

9) $(-7)-(-5)$

10) $(-4)+(-1)$

11) $3-(-1)$

12) $(-1)+(-6)$

13) $6-3$

14) $(-8)+(-1)$

15) $(-5)+3$

16) $(-1)-8$

17) $2-3$

18) $5-7$

19) $(-8)-(-5)$

20) $(-5)+7$

21) $(-2)+(-5)$

22) $1+(-1)$

23) $5-(-6)$

24) $8-(-1)$

25) $(-6)+3$

26) $(-3)+(-1)$

27) $4-7$

28) $7-3$

29) $(-7)+7$

30) $(-3)+(-5)$

Find each product.

31) $(4)(-1)$

45) $(4)(-6)$

33) $(10)(-8)$

32) $(7)(-5)$

35) $(-4)(-2)$

34) $(-7)(-2)$

37) $(-7)(8)$

36) $(-6)(-1)$

39) $(9)(-4)$

38) $(6)(-1)$

41) $(-5)(2)$

40) $(-9)(-7)$

43) $(-5)(4)$

42) $(-2)(-2)$

44) $(-3)(-9)$

Find each quotient.

46) $\frac{30}{-10}$

47) $\frac{-49}{-7}$

48) $\frac{-12}{-4}$

49) $\frac{-2}{-1}$

50) $\frac{30}{6}$

51) $\frac{20}{10}$

52) $\frac{27}{3}$

53) $\frac{-35}{-5}$

54) $\frac{80}{-8}$

55) $\frac{-8}{-2}$

56) $\frac{50}{5}$

57) $\frac{-16}{2}$

58) $\frac{48}{8}$

59) $\frac{60}{-10}$

60) $\frac{54}{-6}$

0.2

Arithmetic - Fractions

Working with fractions is a very important foundation to algebra. Here we will briefly review reducing, multiplying, dividing, adding, and subtracting fractions. As this is a review, concepts will not be explained in detail as other lessons are.

We always like our final answers when working with fractions to be reduced. Reducing fractions is simply done by dividing both the numerator and denominator by the same number. This is shown in the following example

Example 15.

$$\frac{36}{84} \quad \text{Both numerator and denominator are divisible by 4}$$

$$\frac{36 \div 4}{84 \div 4} = \frac{9}{21} \quad \text{Both numerator and denominator are still divisible by 3}$$

$$\frac{9 \div 3}{21 \div 3} = \frac{3}{7} \quad \text{Our Soultion}$$

The previous example could have been done in one step by dividing both numerator and denominator by 12. We also could have divided by 2 twice and then divided by 3 once (in any order). It is not important which method we use as long as we continue reducing our fraction until it cannot be reduced any further.

The easiest operation with fractions is multiplication. We can multiply fractions by multiplying straight across, multiplying numerators together and denominators together.

Example 16.

$$\frac{6}{7} \cdot \frac{3}{5} \quad \text{Multiply numerators across and denominators across}$$

$$\frac{18}{35} \quad \text{Our Solution}$$

When multiplying we can reduce our fractions before we multiply. We can either reduce verticly with a single fraction, or diagonally with several fractions, as long as we use one number from the numerator and one number from the denominator.

Example 17.

$$\frac{25}{24} \cdot \frac{32}{55} \quad \text{Reduce 25 and 55 by dividing by 5. Reduce 32 and 24 by dividing by 8}$$

$$\frac{5}{3} \cdot \frac{4}{11} \quad \text{Multiply numerators across and denominators across}$$

$$\frac{20}{33} \quad \text{Our Solution}$$

Dividing fractions is very similar to multiplying with one extra step. Dividing fraction requires us to first take the reciprical of the second fraction and multiply. Once we do this, the multiplication problem solves just as the previous problem.

Example 18.

$$\frac{21}{16} \div \frac{28}{6} \quad \text{Multiply by the reciprocal}$$

$$\frac{21}{16} \cdot \frac{6}{28} \quad \text{Reduce 21 and 28 by dividing by 7. Reduce 6 and 16 by dividing by 2}$$

$$\frac{3}{8} \cdot \frac{3}{4} \quad \text{Multiply numerators across and denominators across}$$

$$\frac{9}{32} \quad \text{Our Soultion}$$

To add and subtract fractions we will first have to find the least common denominator (LCD). There are several ways to find an LCD. One way is to find the smallest multiple of the largest denominator that you can also divide the small denomiator by.

Example 19.

$$\begin{aligned} \text{Find the LCD of 8 and 12} \quad & \text{Test multiples of 12} \\ 12? \, \frac{12}{8} \quad & \text{Can}'t \text{ divide 12 by 8} \\ 24? \, \frac{24}{8} = 3 \quad & \text{Yes! We can divide 24 by 8!} \\ 24 \quad & \text{Our Soultion} \end{aligned}$$

Adding and subtracting fractions is identical in process. If both fractions already have a common denominator we just add or subtract the numerators and keep the denominator.

Example 20.

$$\frac{7}{8} + \frac{3}{8} \quad \text{Same denominator, add numerators } 7+3$$

$$\frac{10}{8} \quad \text{Reduce answer, dividing by 2}$$

$$\frac{5}{4} \quad \text{Our Solution}$$

While $\frac{5}{4}$ can be written as the mixed number $1\frac{1}{4}$, in algebra we will almost never use mixed numbers. For this reason we will always use the improper fraction, not the mixed number.

Example 21.

$$\frac{13}{6} - \frac{9}{6} \quad \text{Same denominator, subtract numerators } 13-9$$

$$\frac{4}{6} \quad \text{Reduce answer, dividing by } 2$$

$$\frac{2}{3} \quad \text{Our Solution}$$

If the denominators do not match we will first have to identify the LCD and build up each fraction by multiplying the numerators and denominators by the same number so the denominator is built up to the LCD.

Example 22.

$$\frac{5}{6} + \frac{4}{9} \quad \text{LCD is 18.}$$

$$\frac{\mathbf{3}\cdot 5}{\mathbf{3}\cdot 6} + \frac{4\cdot \mathbf{2}}{9\cdot \mathbf{2}} \quad \text{Multiply first fraction by 3 and the second by 2}$$

$$\frac{15}{18} + \frac{8}{18} \quad \text{Same denominator, add numerators, } 15+8$$

$$\frac{23}{18} \quad \text{Our Solution}$$

Example 23.

$$\frac{2}{3} - \frac{1}{6} \quad \text{LCD is 6}$$

$$\frac{2\cdot 2}{2\cdot 3} - \frac{1}{6} \quad \text{Multiply first fraction by 2, the second already has } a \text{ denominator of 6}$$

$$\frac{4}{6} - \frac{1}{6} \quad \text{Same denominator, subtract numerators, } 4-1$$

$$\frac{3}{6} \quad \text{Reduce answer, dividing by 3}$$

$$\frac{1}{2} \quad \text{Our Solution}$$

Practice - Fractions

Simplify each. Leave your answer as an improper fraction.

1) $\frac{42}{12}$

2) $\frac{25}{20}$

3) $\frac{35}{25}$

4) $\frac{24}{9}$

5) $\frac{54}{36}$

6) $\frac{30}{24}$

7) $\frac{45}{36}$

8) $\frac{36}{27}$

9) $\frac{27}{18}$

10) $\frac{48}{18}$

11) $\frac{40}{16}$

12) $\frac{48}{42}$

13) $\frac{63}{18}$

14) $\frac{16}{12}$

15) $\frac{80}{60}$

16) $\frac{72}{48}$

17) $\frac{72}{60}$

18) $\frac{126}{108}$

19) $\frac{36}{24}$

20) $\frac{160}{140}$

Find each product.

21) $(9)(\frac{8}{9})$

22) $(-2)(-\frac{5}{6})$

23) $(2)(-\frac{2}{9})$

24) $(-2)(\frac{1}{3})$

25) $(-2)(\frac{13}{8})$

26) $(\frac{3}{2})(\frac{1}{2})$

27) $(-\frac{6}{5})(-\frac{11}{8})$

28) $(-\frac{3}{7})(-\frac{11}{8})$

29) $(8)(\frac{1}{2})$

30) $(-2)(-\frac{9}{7})$

31) $(\frac{2}{3})(\frac{3}{4})$

32) $(-\frac{17}{9})(-\frac{3}{5})$

33 $(2)(\frac{3}{2})$

34) $(\frac{17}{9})(-\frac{3}{5})$

35) $(\frac{1}{2})(-\frac{7}{5})$

36) $(\frac{1}{2})(\frac{5}{7})$

Find each quotient.

37) $-2 \div \frac{7}{4}$

39) $\frac{-1}{9} \div \frac{-1}{2}$

40) $-2 \div \frac{-3}{2}$

41) $\frac{-3}{2} \div \frac{13}{7}$

42) $\frac{5}{3} \div \frac{7}{5}$

43) $-1 \div \frac{2}{3}$

44) $\frac{10}{9} \div -6$

45) $\frac{8}{9} \div \frac{1}{5}$

46) $\frac{1}{6} \div \frac{-5}{3}$

47) $\frac{-9}{7} \div \frac{1}{5}$

48) $\frac{-13}{8} \div \frac{-15}{8}$

49) $\frac{-2}{9} \div \frac{-3}{2}$

50) $\frac{-4}{5} \div \frac{-13}{8}$

51) $\frac{1}{10} \div \frac{3}{2}$

38) $\frac{-12}{7} \div \frac{-9}{5}$

52) $\frac{5}{3} \div \frac{5}{3}$

Evaluate each expression.

53) $\frac{1}{3} + (-\frac{4}{3})$

54) $\frac{1}{7} + (-\frac{11}{7})$

55) $\frac{3}{7} - \frac{1}{7}$

56) $\frac{1}{3} + \frac{5}{3}$

57) $\frac{11}{6} + \frac{7}{6}$

58) $(-2) + (-\frac{15}{8})$

59) $\frac{3}{5} + \frac{5}{4}$

60) $(7-1) - \frac{2}{3}$

61) $\frac{2}{5} + \frac{5}{4}$

62) $\frac{12}{7} - \frac{9}{7}$

63) $\frac{9}{8} + (-\frac{2}{7})$

64) $(-2) + \frac{5}{6}$

65) $1 + (-\frac{1}{3})$

66) $\frac{1}{2} - \frac{11}{6}$

67) $(-\frac{1}{2}) + \frac{3}{2}$

68) $\frac{11}{8} - \frac{1}{2}$

69) $\frac{1}{5} + \frac{3}{4}$

70) $\frac{6}{5} - \frac{8}{5}$

71) $(-\frac{5}{7}) - \frac{15}{8}$

72) $(-\frac{1}{3}) + (-\frac{8}{5})$

73) $6 - \frac{8}{7}$

74) $(-6) + (-\frac{5}{3})$

75) $\frac{3}{2} - \frac{15}{8}$

76) $(-1) - (-\frac{1}{3})$

77) $(-\frac{15}{8}) + \frac{5}{3}$

78) $\frac{3}{2} + \frac{9}{7}$

79) $(-1) - (-\frac{1}{6})$

80) $(-\frac{1}{2}) - (-\frac{3}{5})$

81) $\frac{5}{3} - (-\frac{1}{3})$

82) $\frac{9}{7} - (-\frac{5}{3})$

0.3

Arithmetic - Order of Operations

When simplifying expressions it is important that we simplify them in the correct order. Consider the following problem done two different ways:

Example 24.

$$\begin{array}{cccc} \underbrace{2+5}\cdot 3 & \text{Add First} & 2+\underbrace{5\cdot 3} & \text{Multiply} \\ \underbrace{7\cdot 3} & \text{Multiply} & \underbrace{2+15} & \text{Add} \\ 21 & \text{Solution} & 17 & \text{Solution} \end{array}$$

The previous example illustrates that if the same problem is done two different ways we will arrive at two different solutions. However, only one method can be correct. It turns out the second method, 17, is the correct method. The order of operations ends with the most basic of operations, addition (or subtraction). Before addition is completed we must do repeated addition or multiplication (or division). Before multiplication is completed we must do repeated multiplication or exponents. When we want to do something out of order and make it come first we will put it in parenthesis (or grouping symbols). This list then is our order of operations we will use to simplify expressions.

Order of Operations:

Parenthesis (Grouping)

Exponents

Multiply and Divide (Left to Right)

Add and Subtract (Left to Right)

Multiply and Divide are on the same level because they are the same operation (division is just multiplying by the reciprocal). This means they must be done left to right, so some problems we will divide first, others we will multiply first. The same is true for adding and subtracting (subtracting is just adding the opposite).

Often students use the word PEMDAS to remember the order of operations, as the first letter of each operation creates the word PEMDAS. However, it is the author's suggestion to think about PEMDAS as a virticle word written as: $\begin{array}{c} \text{P} \\ \text{E} \\ \text{MD} \\ \text{AS} \end{array}$ so we don't forget that multiplication and division are done left to right (same with addition and subtraction). Another way students remember the order of operations is to think of a phrase such as "Please Excuse My Dear Aunt Sally" where each word starts with the same letters as the order of operations start with.

Example 25.

$$\begin{array}{rl}2+3(\underbrace{9-4})^2 & \text{Parenthesis first}\\ 2+3\underbrace{(5)^2} & \text{Exponents}\\ 2+\underbrace{3(25)} & \text{Multiply}\\ \underbrace{2+75} & \text{Add}\\ 77 & \text{Our Solution}\end{array}$$

It is very important to remember to multiply and divide from from left to right!

Example 26.

$$\begin{array}{rl}\underbrace{30\div 3}\cdot 2 & \text{Divide first (left to right!)}\\ \underbrace{10\cdot 2} & \text{Multiply}\\ 20 & \text{Our Solution}\end{array}$$

In the previosu example, if we had multiplied first five would have been the answer which is incorrect.

If there are several parenthesis in a problem we will start with the inner most parenthesis and work our way out. Inside each parenthesis we simplify using the order of operations as well. To make it easier to know which parenthesis goes with which parenthesis, different types of parenthesis will be used such as { } and [ ] and ( ), these parenthesis all mean the same thing, they are parenthesis and must be evaluated first.

Example 27.

$$\begin{array}{rl}2\{8^2-7[32-4(\underbrace{3^2}+1)](-1)\} & \text{Inner most parenthesis, exponents first}\\ 2\{8^2-7[32-4(\underbrace{9+1})](-1)\} & \text{Add inside those parenthesis}\\ 2\{8^2-7[32\underbrace{-4(10)}](-1)\} & \text{Multiply inside inner most parenthesis}\\ 2\{8^2-7[\underbrace{32-40}](-1)\} & \text{Add inside those parenthesis}\\ 2\{8^2\underbrace{-7[-8]}(-1)\} & \text{Multiply left to right, sign with number, } -7[-8]\\ 2\{8^2+\underbrace{56(-1)}\} & \text{Finish multiplying}\\ 2\{\underbrace{8^2}-56\} & \text{Exponents inside parenthesis}\\ 2\{\underbrace{64-56}\} & \text{Subtract inside parenthesis}\\ \underbrace{2\{8\}} & \text{Multiply}\\ 16 & \text{Our Solution}\end{array}$$

As the above example illustrates, it can take several steps to complete a problem. The key to successfully solve order of operations problems is to take the time to show your work and do one step at a time. This will reduce the chance of making a mistake along the way.

There are several types of grouping symbols that can be used besides parenthesis. One type is a fraction bar. If we have a fraction, the entire numerator and the entire denominator must be evaluated before we reduce the fraction. In these cases we can simplify in both the numerator and denominator at the same time.

Example 28.

$$\frac{2^4-(-8)\cdot 3}{15\div 5-1} \quad \text{Exponent in the numerator, divide in denominator}$$

$$\frac{16-(-8)\cdot 3}{5-1} \quad \text{Multiply in the numerator, subtract in denominator}$$

$$\frac{16-(-24)}{4} \quad \text{Add the opposite to simplify numerator, denominator is done.}$$

$$\frac{40}{4} \quad \text{Reduce, divide}$$

$$10 \quad \text{Our Solution}$$

Another type of grouping symbol also has an operation with it, absolute value. When we have absolute value we will evaluate everything inside the absolute value, just as if it were a normal parenthesis. Then once the inside is completed we will take the absolute value, or distance from zero, to make the number positive.

Example 29.

$$1+3|-4^2-(-8)|+2|3+(-5)^2| \quad \text{Evaluate absolute values first, exponents}$$

$$1+3|-16-(-8)|+2|3+25| \quad \text{Add inside absolute values}$$

$$1+3|-8|+2|28| \quad \text{Evaluate absolute values}$$

$$1+3(8)+2(28) \quad \text{Multiply left to right}$$

$$1+24+2(28) \quad \text{Finish multiplying}$$

$$1+24+56 \quad \text{Add left to right}$$

$$25+56 \quad \text{Add}$$

$$81 \quad \text{Our Solution}$$

The above example also illustrates an important point about exponents. Exponents only are considered to be on the number they are attached to. This means when we see -4^2, only the 4 is squared, giving us $-(4^2)$ or -16. But when the negative is in parentheses, such as $(-5)^2$ the negative is part of the number and is also squared giving us a positive solution, 25.

Practice - Order of Operation

Solve.

1) $-6\cdot 4(-1)$

2) $(-6\div 6)^3$

3) $3-(8)\div|4|$

4) $5(-5+6)\cdot 6^2$

5) $8\div 4\cdot 2$

6) $7-5+6$

7) $[-9-(2-5)]\div(-6)$

8) $(-2\cdot 2^3\cdot 2)\div(-4)$

9) $-6+(-3-3)^2\div\left|3\right|$

10) $(-7-5)\div[-2-2-(-6)]$

11) $4-2\left|3^2-16\right|$

12) $\frac{-10-6}{(-2)^2}-5$

13) $[-1-(-5)]|3+2|$

14) $-3-\{3-[-3(2+4)-(-2)]\}$

15) $\frac{2+4|7+2^2|}{4\cdot 2+5\cdot 3}$

16) $-4-[2+4(-6)-4-\left|2^2-5\cdot 2\right|]$

17) $[6\cdot 2+2-(-6)](-5+\left|\frac{-18}{6}\right|)$

18) $2\cdot(-3)+3-6[-2-(-1-3)]$

19) $\frac{-13-2}{2-(-1)^3+(-6)-[-1-(-3)]}$

20) $\frac{-5^2+(-5)^2}{|4^2-2^5|-2\cdot 3}$

21) $6\cdot\frac{-8-4+(-4)-[-4-(-3)]}{(4^2+3^2)\div 5}$

22) $\frac{-9\cdot 2-(3-6)}{1-(-2+1)-(-3)}$

23) $\frac{2^3+4}{4-6+(-4)-[-5(-1)(-5)]}$

24) $\frac{13+(-3)^2+4(-3)+1-[-10-(-6)]}{\{[4+5]\div[4^2-3^3(4-3)-8]\}+2}$

25) $\frac{5+3^2-24\div 6\cdot 2}{[5+3(2^2-5)]+|2^2-5|^2}$

0.4

Arithmetic - Properties of Algebra

In algebra we will often need to simplify an expression to make it easier to use. There are three basic forms of simplifying which we will review here.

The first form of simplifying expressions is used when we know what number each variable in the expression represents. If we know what they represent we can replace each variable with the equivalent number and simplify what remains using order of operations.

Example 30.

$$\begin{array}{rl} p(q+6) \text{ when } p=3 \text{ and } q=5 & \text{Replace } p \text{ with } 3 \text{ and } q \text{ with } 5 \\ (3)\mathbf{((5)+6)} & \text{Evaluate parenthesis} \\ \mathbf{(3)(11)} & \text{Multiply} \\ 33 & \text{Our Solution} \end{array}$$

Whenever a variable is replaced with something, we will put the new number inside a set of parenthesis. Notice the 3 and 5 in the previous example are in parenthesis. This is to preserve operations that are sometimes lost in a simple replacement. Sometimes the parenthesis won't make a difference, but it is a good habit to always use them to prevent problems later.

Example 31.

$$\begin{array}{rl} x+zx(3-z)\left(\frac{x}{3}\right) \text{ when } x=-6 \text{ and } z=-2 & \text{Replace all } x's \text{ with } 6 \text{ and } z's \text{ with } 2 \\ (-6)+(-2)(-6)\mathbf{(3-(-2))}\left(\mathbf{\frac{(-6)}{2}}\right) & \text{Evaluate parenthesis} \\ -6+\mathbf{(-2)(-6)}(5)(-3) & \text{Multiply left to right} \\ -6+\mathbf{12(5)}(-3) & \text{Multiply left to right} \\ -6+\mathbf{60(-3)} & \text{Multiply} \\ \mathbf{-6-180} & \text{Subtract} \\ -186 & \text{Our Solution} \end{array}$$

It will be more common in our study of algebra that we do not know the value of the variables. In this case, we will have to simplify what we can and leave the variables in our final solution. One way we can simplify expressions is to combine like terms. **Like terms** are terms where the variables match exactly (exponents included). Examples of like terms would be $3xy$ and $-7xy$ or $3a^2b$ and $8a^2b$ or -3 and 5. If we have like terms we are allowed to add (or subtract) the numbers in front of the variables, then keep the variables the same. This is shown in the following examples

Example 32.

$$\begin{array}{rl} 5x-2y-8x+7y & \text{Combine like terms } 5x-8x \text{ and } -2y+7y \\ -3x+5y & \text{Our Solution} \end{array}$$

Example 33.

$$\begin{array}{rl} 8x^2-3x+7-2x^2+4x-3 & \text{Combine like terms } 8x^2-2x^2 \text{ and } -3x+4x \text{ and } 7-3 \\ 6x^2+x+4 & \text{Our Solution} \end{array}$$

As we combine like terms we need to interpret subtraction signs as part of the following term. This means if we see a subtraction sign, we treat the following term like a negative term, the sign always stays with the term.

A final method to simplify is known as distributing. Often as we work with problems there will be a set of parenthesis that make solving a problem difficult, if not impossible. To get rid of these unwanted parenthesis we have the distributive property. Using this property we multiply the number in front of the parenthesis by each term inside of the parenthesis.

Distributive Property: $\boldsymbol{a(b+c)=\mathbf{ab}+\mathbf{ac}}$

Several examples of using the distributive property are given below.

Example 34.

$$\begin{array}{rl} 4(2x-7) & \text{Multiply each term by } 4 \\ 8x-28 & \text{Our Solution} \end{array}$$

Example 35.

$$\begin{array}{rl} -7(5x-6) & \text{Multiply each term by } -7 \\ -35+42 & \text{Our Solution} \end{array}$$

In the previous example we again use the fact that the sign goes with the number, this means we treat the -6 as a negative number, this gives $(-7)(-6)=42$, a positive number. The most common error in distributing is a sign error, be very careful with your signs!

It is possible to distribute just a negative through parenthesis. If we have a negative in front of parenthesis we can think of it like a -1 in front and distribute the -1 through. This is shown in the following example.

Example 36.

$$\begin{array}{rl} -(4x-5y+6) & \text{Negative can be thought of as} -1 \\ -1(4x-5y+6) & \text{Multiply each term by} -1 \\ -4x+5y-6 & \text{Our Solution} \end{array}$$

Distributing through parenthesis and combining like terms can be combined into one problem. Order of operations tells us to multiply (distribute) first then add or subtract last (combine like terms). Thus we do each problem in two steps, distribute then combine.

Example 37.

$$\begin{array}{rl} 5+3(2x-4) & \text{Distribute 3, multipling each term} \\ 5+6x-12 & \text{Combine like terms } 5-12 \\ -7+6x & \text{Our Solution} \end{array}$$

Example 38.

$$\begin{array}{rl} 3x-2(4x-5) & \text{Distribute} -2\text{, multilpying each term} \\ 3x-8x+10 & \text{Combine like terms } 3x-8x \\ -5x+10 & \text{Our Solution} \end{array}$$

In the previous example we distributed -2, not just 2. This is because we will always treat subtraction like a negative sign that goes with the number after it. This makes a big difference when we multiply by the -5 inside the parenthesis, we now have a positive answer. Following are more involved examples of distributing and combining like terms.

Example 39.

$$\begin{array}{rl} 2(5x-8)-6(4x+3) & \text{Distribute 2 into first parenthesis and} -6 \text{ into second} \\ 10x-16-24x-12 & \text{Combine like terms } 10x-24x \text{ and } -16-12 \\ -12x-28 & \text{Our Solution} \end{array}$$

Example 40.

$$\begin{array}{rl} 4(3x-8)-(2x-7) & \text{Negative (subtract) in middle can be thought of as} -1 \\ 4(3x-8)-1(2x-7) & \text{Distribute 4 into first parenthesis,} -1 \text{ into second} \\ 12x-32-2x+7 & \text{Combine like terms } 12x-2x \text{ and } -32+7 \\ 10x-25 & \text{Our Solution} \end{array}$$

Practice - Properties of Algebra

Evaluate each using the values given.

1) $p+1+q-m$; use $m=1, p=3, q=4$

2) y^2+y-z; use $y=5, z=1$

3) $p-\frac{pq}{6}$; use $p=6$ and $q=5$

4) $\frac{6+z-y}{3}$; use $y=1, z=4$

5) $c^2-(a-1)$; use $a=3$ and $c=5$

6) $x+6z-4y$; use $x=6, y=4, z=4$

7) $5j+\frac{kh}{2}$; use $h=5, j=4, k=2$

8) $5(b+a)+1+c$; use $a=2, b=6, c=5$

9) $\frac{4-(p-m)}{2}+q$; use $m=4, p=6, q=6$

10) $z+x-(1^2)^3$; use $x=5, z=4$

11) $m+n+m+\frac{n}{2}$; use $m=1$ and $n=2$

12) $3+z-1+y-1$; use $y=5, z=4$

13) $q-p-(q-1-3)$; use $p=3, q=6$

14) $p+(q-r)(6-p)$; use $p=6$, $q=5$, $r=5$

15) $y-(4-y-(z-x)$; use $x=3$, $y=1$, $z=6$

16) $4z-(x+x-(z-z))$; use $x=3$, $z=2$

17) $k\times 3^2-(j+k)-5$; use $j=4, k=5$

18) $a^3(c^2-c)$; use $a=3, c=2$

19) $zx-(z-\frac{4+x}{6})$; use $x=2, z=6$

20) $5+qp+pq-q$; use $p=6, q=3$

Combine Like Terms

21) $r-9+10$

22) $-4x+2-4$

23) $n+m$

24) $4b+6+1+7b$

25) $8v+7v$

26) $-x+8x$

27) $-7x-2x$

28) $-7a-6+5$

29) $k-2+7$

30) $-8p+5p$

31) $x-10-6x+1$

32) $1-10n-10$

33) $x-10+2m$

34) $1-r-6$

35) $9n-1+n+4$

36) $-4b+9b$

Distribute

37) $-8(x-4)$

39) $8n(n+9)$

40) $-(-5+9a)$

41) $7k(-k+6)$

42) $10x(1+2x)$

43) $-6(1+6x)$

44) $-2(n+1)$

45) $8m(5-m)$

46) $-2p(9p-1)$

47) $-9x(4-x)$

48) $4(8n-2)$

49) $-9b(b-10)$

50) $-4(1+7r)$

51) $-8n(5+10n)$

38) $3(8v+9)$

52) $2x(8x-10)$

Simplify.

53) $9(b+10)+5b$

54) $4v-7(1-8v)$

55) $-3x(1-4x)-4x^2$

56) $-8x+9(-9x+9)$

57) $-4k^2-8k\,(8k+1)$

58) $-9-10(1+9a)$

59) $1-7(5+7p)$

60) $-10(x-2)-3$

61) $-10-4(n-5)$

62) $-6(5-m)+3m$

63) $4(x+7)+8(x+4)$

64) $-2r(1+4r)+8r(-r+4)$

65) $-8(n+6)-8n(n+8)$

66) $9(6b+5)-4b(b+3)$

67) $7(7+3v)+10(3-10v)$

68) $-7(4x-6)+2(10x-10)$

69) $2n(-10n+5)-7(6-10n)$

70) $-3(4+a)+6a(9a+10)$

71) $5(1-6k)+10(k-8)$

72) $-7(4x+3)-10(10x+10)$

73) $(8n^2-3n)-(5+4n^2)$

74) $(7x^2-3)-(5x^2+6x)$

75) $(5p-6)+(1-p)$

76) $(3x^2-x)-(7-8x)$

77) $(2-4v^2)+(3v^2+2v)$

78) $(2b-8)+(b-7b^2)$

79) $(4-2k^2)+(8-2k^2)$

80) $(7a^2+7a)-(6a^2+4a)$

81) $(x^2-8)+(2x^2-7)$

82) $(3-7n^2)+(6n^2+3)$

Answers - Integers

1) -2
2) 5
3) 2
4) 2
5) -6
6) -5
7) 8
8) 0
9) -2
10) -5
11) 4
12) -7
13) 3
14) -9
15) -2
16) -9
17) -1
18) -2
19) -3
20) 2
21) -7
22) 0
23) 11
24) 9
25) -3
26) -4
27) -3
28) 4
29) 0
30) -8
31) -4
32) -35
33) -80
34) 14
35) 8
36) 6
37) -56
38) -6
39) -36
40) 63
41) -10
42) 4
43) -20
44) 27
45) -24
46) -3
47) 7
48) 3
49) 2
50) 5
51) 2
52) 9
53) 7
54) -10
55) 4
56) 10
57) -8
58) 6
59) -6
60) -9

Answers - Fractions

1) $\frac{7}{2}$
2) $\frac{5}{4}$
3) $\frac{7}{5}$
4) $\frac{8}{3}$
5) $\frac{3}{2}$
6) $\frac{5}{4}$
7) $\frac{5}{4}$
8) $\frac{4}{3}$
9) $\frac{3}{2}$
10) $\frac{8}{3}$
11) $\frac{5}{2}$
12) $\frac{8}{7}$
13) $\frac{7}{2}$
14) $\frac{4}{3}$

15) $\frac{4}{3}$

16) $\frac{3}{2}$

17) $\frac{6}{5}$

18) $\frac{7}{6}$

19) $\frac{3}{2}$

20) $\frac{8}{7}$

21) 8

22) $\frac{5}{3}$

23) $-\frac{4}{9}$

24) $-\frac{2}{3}$

25) $-\frac{13}{4}$

26) $\frac{3}{4}$

27) $\frac{33}{20}$

28) $\frac{33}{56}$

29) 4

30) $\frac{18}{7}$

31) $\frac{1}{2}$

32) $-\frac{19}{20}$

33) 3

34) $-\frac{17}{15}$

35) $-\frac{7}{10}$

36) $\frac{5}{14}$

37) $-\frac{8}{7}$

38) $\frac{20}{21}$

39) $\frac{2}{9}$

40) $\frac{4}{3}$

41) $-\frac{21}{26}$

42) $\frac{25}{21}$

43) $-\frac{3}{2}$

44) $-\frac{5}{27}$

45) $\frac{40}{9}$

46) $-\frac{1}{10}$

47) $-\frac{45}{7}$

48) $\frac{13}{15}$

49) $\frac{4}{27}$

50) $\frac{32}{65}$

51) $\frac{1}{15}$

52) 1

53) ·

54) ·

55) $\frac{2}{7}$

56) 2

57) 3

58) $-\frac{31}{8}$

59) $\frac{37}{20}$

60) $-\frac{5}{3}$

61) $\frac{33}{20}$

62) $\frac{3}{7}$

63) $\frac{47}{56}$

64) $-\frac{7}{6}$

65) $\frac{2}{3}$

66) $-\frac{4}{3}$

67) 1

68) $\frac{7}{8}$

69) $\frac{19}{20}$

70) $-\frac{2}{5}$

71) $-\frac{145}{56}$

72) $-\frac{29}{15}$

73) $\frac{34}{7}$

74) $-\frac{23}{3}$

75) $-\frac{3}{8}$

76) $-\frac{2}{3}$

77) $-\frac{5}{24}$

78) $\frac{39}{14}$

79) $-\frac{5}{6}$

80) $\frac{1}{10}$

81) 2

82) $\frac{62}{21}$

Answers - Order of Operation

1) 24

2) -1

3) 5

4) 180

5) 4

6) 8

7) 1

8) 8

9) 6

10) -6

11) -10

12) -9

13) 20

14) -16

15) 2

16) 26

17) -40

18) -15

19) 3

20) 0

21) -18

22) -3

23) -4

24) 3

25) 2

Answers - Properties of Algebra

1) 7

2) 29

3) 1

4) 3

5) 23

6) 14

7) 25

8) 46

9) 7

10) 8

11) 5

12) 10

13) 1

14) 6

15) 1

16) 2

17) 31

18) 54

19) 7

20) 38

21) $r+1$

22) $-4x-2$

23) $2n$

24) $11b+7$

25) $15v$

26) $7x$

27) $-9x$

28) $-7a-1$

29) $k+5$

30) $-3p$

31) $-5x-9$

32) $-9-10n$

33) $-m$

34) $-5-r$

35) $10n+3$

36) $5b$

37) $-8x+32$

38) $24v+27$

39) $8n^2+72n$

40) $5-9a$

41) $-7k^2+42k$

42) $10x+20x^2$

43) $-6-36x$

44) $-2n-2$

45) $40m-9m^2$

46) $-18p^2+2p$

47) $-36x+9x^2$

48) $32n-8$

49) $-9b^2+90b$

50) $-4-28r$

51) $-40n-80n^2$

52) $16x^2-20x$

53) $14b+90$

54) $60v-7$

55) $-3x+8x^2$

56) $-89x+81$

57) $-68k^2-8k$

58) $-19-90a$

59) $-34-49p$

60) $-10x+17$

61) $10-4n$

62) $-30+9m$

63) $12x+60$

64) $30r-16r^2$

65) $-72n-48-8n^2$

66) $-66b-45-4b^2$

67) $79-79v$

68) $-8x+22$

69) $-20n^2+80n-42$

70) $-12+57a+54a^2$

71) $-75-20k$

72) $-128x-121$

73) $4n^2-3n-5$

74) $2x^2-6x-3$

75) $4p-5$

76) $3x^2+7x-7$

77) $-v^2+2v+2$

78) $-7b^2+3b-8$

79) $-4k^2+12$

80) a^2+3a

81) $3x^2-15$

82) $-n^2+6$