9.7

1) In a landscape plan, a rectangular flowerbed is designed to be 4 meters longer than it is wide. If 60 square meters are needed for the plants in the bed, what should the dimensions of the rectangular bed be?

\[
x(x + 4) = 60 \\
x^2 + 4x = 60 \\
-60 - 60 \\
x^2 + 4x - 60 = 0 \\
(x - 6)(x + 10) = 0 \\
x - 6 = 0 \quad x + 10 = 0 \\
\frac{+6}{6} \quad \frac{+6}{6} \quad -10 \quad -10 \\
x = 6 \\
\]

6m by 10m

3) A rectangular lot is 20 yards longer than it is wide and its area is 2400 square yards. Find the dimensions of the lot.

\[
x(x + 20) = 2400 \\
x^2 + 20x = 2400 \\
\left(20 \cdot \frac{1}{2}\right)^2 = 10^2 = 100 \\
x^2 + 20x + 100 = 2400 + 100 \\
\sqrt{(x+10)^2} = \sqrt{2500} \\
x + 10 = \pm 50 \\
-10 \quad -10 \\
x = -10 \pm 50 \\
x = 40, -60 \\
\]

40yds x 60 yds

5) The length of a rectangular lot is 4 rods greater than its width, and its area is 60 square rods. Find the dimensions of the lot.

\[
x(x + 4) = 60 \\
x^2 + 4x = 60 \\
-60 - 60 \\
x^2 + 4x - 60 = 0 \\
(x + 10)(x - 6) = 0 \\
x + 10 = 0 \quad x - 6 = 0 \\
-10 - 10 \quad +6 +6 \\
x = 6 \\
\]

6 rods x 10 rods
7) A rectangular piece of paper is twice as long as a square piece and 3 inches wider. The area of the rectangular piece is 108 in². Find the dimensions of the square piece.

\[2x(x + 3) = 108\]
\[2x^2 + 6x = 108\]
\[-108 - 108\]
\[\frac{2x^2}{2} + \frac{6x}{2} - \frac{108}{2} = \frac{0}{2}\]
\[x^2 + 3x - 54 = 0\]
\[(x + 9)(x - 6) = 0\]
\[x + 9 = 0 \quad x - 6 = 0\]
\[-9 - 9 + 6 + 6\]
\[x = 6\]

6in x 6in

9) The area of a rectangle is 48 ft² and its perimeter is 32 ft. Find its length and width.

\[2x + 2y = 32\]
\[-2x - 2x\]
\[\frac{2y}{2} = \frac{32}{2} - \frac{2x}{2}\]
\[y = 16 - x\]
\[xy = 48\]
\[x(16 - x) = 48\]
\[16x - x^2 = 48\]
\[-16x + x^2 - 16x + x^2\]
\[0 = x^2 - 16x + 48\]
\[0 = (x - 12)(x - 4)\]
\[x - 12 = 0 \quad x - 4 = 0\]
\[+12 +12 +4 +4\]
\[x = 12 \quad x = 4\]

12ft x 4 ft

\[y = 16 - 12 = 4 \quad y = 16 - 4 = 12\]
11) A mirror 14 inches by 15 inches has a frame of uniform width. If the area of the frame equals that of the mirror, what is the width of the frame?

\[
(15 + 2x)(14 + 2x) = 420 \\
210 + 30x + 28 + 4x^2 = 420 \\
4x^2 + 58x + 210 = 420
\]

\[
\frac{4x^2}{2} + \frac{58x}{2} - \frac{210}{2} = \frac{0}{2} \\
2x^2 + 29x - 105 = 0 \\
(2x + 35)(x - 3) = 0 \\
2x + 35 = 0 \quad x - 3 = 0
\]

\[
A = 2(14 \cdot 15) = 420 \\
\frac{2x}{2} = \frac{-35}{2} \quad x = 3
\]

3 in

13) A grass plot 9 yards long and 6 yards wide has a path of uniform width around it. If the area of the path is equal to the area of the plot, determine the width of the path.

\[
(6 + 2x)(9 + 2x) = 108 \\
54 + 12x + 18x + 4x^2 = 108 \\
4x^2 + 30x + 54 = 108
\]

\[
\frac{4x^2}{2} + \frac{30x}{2} - \frac{54}{2} = \frac{0}{2} \\
2x^2 + 15x - 27 = 0 \\
(2x - 3)(x + 9) = 0 \\
2x - 3 = 0 \quad x + 9 = 0
\]

\[
A = 2(9 \cdot 6) = 108 \\
\frac{2x}{2} = \frac{3}{2} \quad x = \frac{3}{2} = 1.5
\]

1.5 yds

15) A page is to have a margin of 1 inch, and is to contain 35 in\(^2\) of painting. How large must the page be if the length is to exceed the width by 2 inches?

\[
x(x - 2) = 35 \\
x^2 - 2x = 35 \\
-35 - 35 \\
x^2 - 2x - 35 = 0 \\
(x - 7)(x + 5) = 0 \\
x - 7 = 0 \quad x + 5 = 0
\]

\[
\frac{+7 + 7}{x} = \frac{-5 - 5}{x} \\
x = 7 \quad x = -5
\]

7in x 9in
17) A rectangular wheat field is 80 rods long by 60 rods wide. A strip of uniform width is cut around the field, so that half the grain is left standing in the form of a rectangular plot. How wide is the strip that is cut?

\[
(80 - 2x)(60 - 2x) = 2400
\]

\[
48000 - 160x - 120x + 4x^2 = 2400
\]

\[
4x^2 - 280x + 4800 = 2400
\]

\[
x^2 - 70x + 600 = 0
\]

\[
(x - 10)(x - 60) = 0
\]

\[
x - 10 = 0 \quad x - 60 = 0
\]

\[
\begin{align*}
x + 10 + 10 + 60 + 60 \\
\end{align*}
\]

\[
x = 10 \quad x = 60
\]

10 rods

19) A rectangular field 225 ft by 120 ft has a ring of uniform width cut around the outside edge. The ring leaves 65% of the field uncut in the center. What is the width of the ring?

\[
(225 - 2x)(120 - 2x) = 17500
\]

\[
27000 - 450x - 240x + 4x^2 = 17500
\]

\[
4x^2 - 690x + 27000 = 17500
\]

\[
x^2 - 17500 - 17500
\]

\[
\begin{align*}
x^2 - 690x + 9450 &= 0 \\
\frac{4x^2}{2} - \frac{690x}{2} + \frac{9450}{2} &= 0 \\
2x^2 - 345x + 4725 &= 0
\end{align*}
\]

\[
A = 0.65(120 \cdot 225)17500
\]

\[
\frac{345\pm\sqrt{345^2-4(2)(4725)}}{2(2)} = \frac{345\pm\sqrt{81225}}{4} = \frac{345\pm285}{2} = 15 &\text{ & 15}
\]

15 ft
21) A frame is 15 in by 25 in and is of uniform width. The inside of the frame leaves 75% of the total area available for the picture. What is the width of the frame?

\[ (15 - 2x)(25 - 2x) = 281.25 \]
\[ 375 - 30x - 50x + 4x^2 = 281.25 \]
\[ 4x^2 - 80x + 375 = 281.25 \]
\[ -281.25 - 281.25 \]
\[ \frac{4x^2}{4} - \frac{80x}{4} + \frac{93.75}{4} = \frac{0}{4} \]
\[ x^2 - 20x + 23.4375 = 0 \]
\[ -23.4375 - 23.4375 \]
\[ x^2 - 20x = -23.4375 \]
\[ A = .75(25 \cdot 15) = 281.25 \]
\[ (-20 \cdot \frac{1}{2})^2 = (-10)^2 = 100 \]
\[ x^2 - 20x + 100 = -23.4375 + 100 \]
\[ \sqrt{(x - 10)^2} = \sqrt{76.5625} \]
\[ x - 10 = \pm 8.75 \]
\[ +10 -10 \]
\[ x = 10 \pm 8.75 \]
\[ x = 18.75, 1.25 \]

1.25 in

23) The farmer in the previous problem has a neighbor who has a field 325 ft by 420 ft. His neighbor wants to increase the size of his field by 20% by cultivating a band of uniform width around the outside of his lot. How wide a band should his neighbor cultivate?

\[ (420 + 2x)(325 + 2x) = 163800 \]
\[ 136500 + 840x + 650x + 4x^2 = 163800 \]
\[ 4x^2 + 1490x + 136500 = 163800 \]
\[ -163800 - 163800 \]
\[ \frac{4x^2}{4} + \frac{1490x}{4} - \frac{27300}{4} = \frac{0}{4} \]
\[ x^2 + 372.5x - 6825 = 0 \]
\[ \frac{-745 \pm \sqrt{745^2 - 4(2)(-13650)}}{2(2)} \]
\[ \frac{-745 \pm \sqrt{564225}}{4} \]
\[ \frac{-745 \pm 815}{4} = 17.5, -390 \]

17.5 ft
25) Donna has a garden that is 30 ft by 36 ft. She wants to increase the size of the garden by 40%. How wide a ring around the outside should she cultivate?

\[(36 + 2x)(30 + 2x) = 1512\]

\[
\begin{align*}
1080 + 72x + 60x + 4x^2 &= 1512 \\
4x^2 + 132x + 1080 &= 1512 \\
-1512 &= -1512 \\
\end{align*}
\]

\[
\frac{4x^2 + 132x - 432}{4} = \frac{0}{4}
\]

\[
x^2 + 33x - 108 = 0
\]

\[
-33 \pm \sqrt{33^2 - 4(1)(-108)} = -33 \pm \sqrt{1321} = \frac{-33 \pm 39}{2} = 3, -\infty
\]

3 ft.