

Quadratics - Simultaneous Products

Objective: Solve simultaneous product equations using substitution to create a rational equation.

When solving a system of equations where the variables are multiplied together

we can use the same idea of substitution that we used with linear equations. When we do so we may end up with a quadratic equation to solve. When we used substitution we solved for a variable and substitute this expression into the other equation. If we have two products we will choose a variable to solve for first and divide both sides of the equations by that variable or the factor containing the variable. This will create a situation where substitution can easily be done.

Example 1.

$xy = 48$		
$(x + 3)(y - 2) = 54$		To solve for x , divide first equation by x , second by $x + 3$
$y = \frac{48}{x}$ and $y - 2 = \frac{54}{x + 3}$		Substitute $\frac{48}{x}$ for y in the second equation
$\frac{48}{x} - 2 = \frac{54}{x + 3}$		Multiply each term by LCD: $x(x + 3)$
$\frac{48x(x + 3)}{x} - 2x(x + 3) = \frac{54x(x + 3)}{x + 3}$		Reduce each fraction
$48(x + 3) - 2x(x + 3) = 54x$		Distribute
$48x + 144 - 2x^2 - 6x = 54x$		Combine like terms
$-2x^2 + 42x + 144 = 54x$		Make equation equal zero
$\quad \quad \quad \underline{-54x} \quad \quad \quad \underline{-54x}$		Subtract $54x$ from both sides
$-2x^2 - 12x + 144 = 0$		Divide each term by GCF of -2
$\quad \quad \quad x^2 + 6x - 72 = 0$		Factor
$\quad \quad \quad (x - 6)(x + 12) = 0$		Set each factor equal to zero
$x - 6 = 0$ or $x + 12 = 0$		Solve each equation
$\quad \quad \quad \underline{+6 + 6} \quad \quad \quad \underline{-12 - 12}$		
$\quad \quad \quad x = 6$ or $x = -12$		Substitute each solution into $xy = 48$
$6y = 48$ or $-12y = 48$		Solve each equation
$\quad \quad \quad \underline{6} \quad \underline{6} \quad \quad \quad \underline{-12} \quad \underline{-12}$		
$\quad \quad \quad y = 8$ or $y = -4$		Our solutions for y ,
$(6, 8)$ or $(-12, -4)$		Our Solutions as ordered pairs

These simultaneous product equations will also solve by the exact same pattern. We pick a variable to solve for, divide each side by that variable, or factor containing the variable. This will allow us to use substitution to create a rational expression we can use to solve. Quite often these problems will have two solu-