9.6

Quadratics - Rectangles

An application of solving quadratic equations comes from the formula for the area of a rectangle. The area of a rectangle can be calculated by multiplying the width by the length. To solve problems with rectangles we will first draw a picture to represent the problem and use the picture to set up our equation.

Example 1.

The length of a rectangle is 3 more than the width. If the area is 40 square inches, what are the dimensions?

$\boxed{40}\ x$	We do not know the width, x.
$x+3$	Length is 4 more, or $x+4$, and area is 40.
$x(x+3)=40$	Multiply length by width to get area
$x^2+3x=40$	Distribute
$\underline{-40\quad -40}$	Make equation equal zero
$x^2+3x-40=0$	Factor
$(x-5)(x+8)=0$	Set each factor equal to zero
$x-5=0$ or $x+8=0$	Solve each equation
$\underline{+5+5}\qquad\underline{-8-8}$	
$x=5$ or $x=-8$	Our x is *a* width, can$'t$ be negative.
$(5)+3=8$	Length is $x+3$, substitute 5 for x to find length
5 in by 8 in	Our Solution

The above rectangle problem is very simple as there is only one rectangle involved. When we compare two rectangles, we may have to get a bit more creative.

Example 2.

If each side of a square is increased by 6, the area is multiplied by 16. Find the side of the original square.

$\boxed{x^2}\ x$	Square has all sides the same length
x	Area is found by multiplying length by width

$\boxed{16x^2}\ x+6$	Each side is increased by 6,
$x+6$	Area is 16 times original area
$(x+6)(x+6)=16x^2$	Multiply length by width to get area
$x^2+12x+36=16x^2$	FOIL
$\underline{-16x^2 \qquad -16x^2}$	Make equation equal zero
$-15x^2+12x+36=0$	Divide each term by -1, changes the signs
$15x^2-12x-36=0$	Solve using the quadratic formula
$x=\frac{12\pm\sqrt{(-12)^2-4(15)(-36)}}{2(15)}$	Evaluate
$x=\frac{16\pm\sqrt{2304}}{30}$	
$x=\frac{16\pm 48}{30}$	Can$'t$ have a negative solution, we will only add
$x=\frac{60}{30}=2$	Our x is the original square
2	Our Solution

Example 3.

The length of a rectangle is 4 ft greater than the width. If each dimension is increased by 3, the new area will be 33 square feet larger. Find the dimensions of the original rectangle.

$\boxed{x(x+4)}\ x$	We don$'t$ know width, x, length is 4 more, $x+4$
$x+4$	Area is found by multiplying length by width
$\boxed{x(x+4)+33}\ x+3$	Increase each side by 3. width becomes $x+3$, length $x+4+3=x+7$
$x+7$	Area is 33 more than original, $x(x+4)+33$
$(x+3)(x+7)=x(x+4)+33$	Set up equation, length times width is area
$x^2+10x+21=x^2+4x+33$	Subtract x^2 from both sides
$\underline{-x^2 \qquad -x^2}$	
$10x+21=4x+33$	Move variables to one side
$\underline{-4x \qquad -4x}$	Subtract $4x$ from each side
$6x+21=33$	Subtract 21 from both sides
$\underline{-21 \quad -21}$	
$6x=12$	Divide both sides by 6
$\frac{}{6}\ \frac{}{6}$	
$x=2$	x is the width of the original

$(2)+4=6$	$x+4$ is the length. Substitue 2 to find
2 ft by 6 ft	Our Solution

From one rectangle we can find two equations. Perimeter is found by adding all the sides of a polygon together. A rectangle as two widths and two lengths, both the same size. So we can use the equation $P = 2l + 2w$ (twice the length plus twice the width).

Example 4.

The area of a rectangle is 168 cm^2. The perimeter of the same rectangle is 52 cm. What are the dimensions of the rectangle?

$\square\, x$	We don't know anything about length or width
y	Use two variables, x and y
$xy=168$	Length times width gives the area.
$2x+2y=52$	Also use perimeter formula.
$\underline{-2x \qquad -2x}$	Solve by substitution, isolate y
$2y=-2x+52$	Divide each term by 2
$\overline{2} \quad \overline{2} \quad \overline{2}$	
$y=-x+26$	Substitute into area equation
$x(-x+26)=168$	Distribute
$-x^2+26x=168$	Divide each term by -1, changing all the signs
$x^2-26x=-168$	Solve by completing the square.
$\left(\frac{1}{2}\cdot 26\right)^2=13^2=169$	Find number to complete the square: $\left(\frac{1}{2}\cdot b\right)^2$
$x^2-26x+324=1$	Add 169 to both sides
$(x-13)^2=1$	Factor
$x-13=\pm 1$	Square root both sides
$\underline{+13 \quad +13}$	
$x=13\pm 1$	Evaluate
$x=14$ or 12	Two options for first side.
$y=-(14)+26=12$	Substitute 14 into $y=-x+26$
$y=-(12)+26=14$	Substitute 12 into $y=-x+26$
	Both are the same rectangle, variables switched!
12 cm by 14 cm	Our Solution

9.6

Practice - Rectangles

1) In a landscape plan, a rectangular flowerbed is designed to be 4 meters longer than it is wide. If 60 square meters are needed for the plants in the bed, what should the dimensions of the rectangular bed be?

2) If the side of a square is increased by 5 the area is multiplied by 4. Find the side of the original square.

3) A rectangular lot is 20 yards longer than it is wide and its area is 2400 square yards. Find the dimensions of the lot.

4) The length of a room is 8 ft greater than its width. If each dimension is increased by 2 ft, the area will be increased by 60 sq. ft. Find the dimensions of the rooms.

5) The length of a rectangular lot is 4 rods greater than its width, and its area is 60 square rods. Find the dimensions of the lot.

6) The length of a rectangle is 15 ft greater than its width. If each dimension is decreased by 2 ft, the area will be decreased by 106 ft^2. Find the dimensions.

7) A rectangular piece of paper is twice as long as a square piece and 3 inches wider. The area of the rectangular piece is 108 in^2. Find the dimensions of the square piece.

8) A room is one yard longer than it is wide. At 75¢ per sq. yd. a covering for the floor costs $31.50. Find the dimensions of the floor.

9) The area of a rectangle is 48 ft^2 and its perimeter is 32 ft. Find its length and width.

10) The dimensions of a picture inside a frame of uniform width are 12 by 16 inches. If the whole area (picture and frame) is 288 in^2, what is the width of the frame?

11) A mirror 14 inches by 15 inches has a frame of uniform width. If the area of the frame equals that of the mirror, what is the width of the frame.

12) A lawn is 60 ft by 80 ft. How wide a strip must be cut around it when mowing the grass to have cut half of it.

13) A grass plot 9 yards long and 6 yards wide has a path of uniform width around it. If the area of the path is equal to the area of the plot, determine the width of the path.

15) A page is to have a margin of 1 inch, and is to contain 35 in^2 of painting. How large must the page be if teh length is to exceed the width by 2 inches?

16) A picture 10 inches long by 8 inches wide has a frame whose area is one half the area of the picture. What are the outside dimensions of the frame?

17) A rectangular wheat field is 80 rods long by 60 rods wide. A strip of uniform width is cut around the field, so that half the grain is left standing in the form of a rectangular plot. How wide is the strip that is cut?

18) A picture 8 inches by 12 inches is placed in a frame of uniform width. If the area of the frame equals the area of the picture find the width of the frame.

19) A rectangular field 225 ft by 120 ft has a ring of uniform width cut around the outside edge. The ring leaves 65% of the field uncut in the center. What is the width of the ring?

20) One Saturday morning George goes out to cut his lot that is 100 ft by 120 ft. He starts cutting around the outside boundary spiraling around towards the center. By noon he has cut 60% of the lawn. What is the width of the ring that he has cut?

21) A frame is 15 in by 25 in and is of uniform width. The inside of the frame leaves 75% of the total area available for the picture. What is the width of the frame?

22) A farmer has a field 180 ft by 240 ft. He wants to increase the area of the field by 50% by cultivating a band of uniform width around the outside. How wide a band should he cultivate?

23) The farmer in the previous problem has a neighber who has a field 325 ft by 420 ft. His neighbor wants to increase the size of his field by 20% by cultivating a band of uniform width around the outside of his lot. How wide a band should his neighbor cultivate?

24) A third farmer has a field that is 500 ft by 550 ft. He wants to increase his field by 20%. How wide a ring should he cultivate around the outside of his field?

25) Donna has a garden that is 30 ft by 36 ft. She wants to increase the size of the garden by 40%. How wide a ring around teh outside should she cultivate?

26) A picture is 12 in by 25 in and is surrounded by a frame of uniform width. The area of the fram is 30% of the area of the picture. How wide is the fram?

27) A landscape architect is designing a rectangular flowerbed to be borded with 28 plants taht are placed 1 meter apart. He needs an inner rectangular space in the center for plants that must be 1 meter from the border of the bed and that require 24 square meters for planting. What should the overall dimensions of the flowerbed be?

Answers - Rectangles

1) 6 m x 10 m
2) 5
3) 40 yd x 60 yd
4) 10 ft x 18 ft
5) 6 x 10
6) 20 ft x 35 ft
7) 6" x 6"
8) 6 yd x 7 yd
9) 4 ft x 12 ft
10) 1.54 in
11) 3 in
12) 10 ft
13) 1.5 yd
14) 7 x 9
15) 1 in
16) 10 rods
17) 2 in
18) 15 ft
19) 60 ft
20) 20 ft
21) 1.25 in
22) 23.16 ft
23) 17.5 ft
24) 25 ft
25) 3 ft
26) 1.145 in
27) 6 m x 8 m