# Radicals - Mixed Index

Knowing that a radical has the same properties as exponents (written as a ratio) allows us to manipulate radicals in new ways. One thing we are allowed to do is reduce, not just the radicand, but the index as well. This is shown in the following example.

**Example 1.**

| | |
|---|---|
| $\sqrt[8]{x^6y^2}$ | Rewrite as raitonal exponent |
| $(x^6y^2)^{\frac{1}{5}}$ | Multiply exponents |
| $x^{\frac{6}{8}}y^{\frac{2}{8}}$ | Reduce each fraction |
| $x^{\frac{3}{4}}y^{\frac{1}{4}}$ | All exponents have denominator of 4, this is our new index |
| $\sqrt[4]{x^3y}$ | Our Solution |

What we have done is reduced our index by dividing the index and all the exponents by the same number (2 in the previous example). If we notice a common factor in the index and all the exponnets on every factor we can reduce by dividing by that common factor. This is shown in the next example

**Example 2.**

| | |
|---|---|
| $\sqrt[24]{a^6b^9c^{15}}$ | Index and all exponents are divisible by 3 |
| $\sqrt[8]{a^2b^3c^5}$ | Our Solution |

We can use the same process when there are coefficients in the problem. We will first write the coefficient as an exponential expression so we can divide the exponet by the common factor as well.

**Example 3.**

| | |
|---|---|
| $\sqrt[9]{8m^6n^3}$ | Write 8 as $2^3$ |
| $\sqrt[9]{2^3m^6n^3}$ | Index and all exponents are divisible by 3 |
| $\sqrt[3]{2m^2n}$ | Our Solution |

We can use a very similar idea to also multiply radicals where the index does not match. First we will consider an example using rational exponents, then identify the pattern we can use.

**Example 4.**

| | |
|---|---|
| $\sqrt[3]{ab^2}\,\sqrt[4]{a^2b}$ | Rewrite as rational exponents |
| $(ab^2)^{\frac{1}{3}}(a^2b)^{\frac{1}{4}}$ | Multiply exponents |
| $a^{\frac{1}{3}}b^{\frac{2}{3}}a^{\frac{2}{4}}b^{\frac{1}{4}}$ | To have one radical need $a$ common denominator, 12 |
| $a^{\frac{4}{12}}b^{\frac{8}{12}}a^{\frac{6}{12}}b^{\frac{3}{12}}$ | Write under $a$ single radical with common index, 12 |
| $\sqrt[12]{a^4b^8a^6b^3}$ | Add exponents |
| $\sqrt[12]{a^{10}b^{11}}$ | Our Solution |

To combine the radicals we need a common index (just like the common denominator). We will get a common index by multiplying each index and exponent by an integer that will allow us to build up to that desired index. This process is shown in the next example.

**Example 5.**

| | |
|---|---|
| $\sqrt[4]{a^2b^3}\,\sqrt[6]{a^2b}$ | Common index is 12. |
| | Multiply first index and exponents by 3, second by 2 |
| $\sqrt[12]{a^6b^9a^4b^2}$ | Add exponents |
| $\sqrt[12]{a^{10}b^{11}}$ | Our Solution |

Often after combining radicals of mixed index we will need to simplify the resulting radical.

**Example 6.**

| | |
|---|---|
| $\sqrt[5]{x^3y^4}\,\sqrt[3]{x^2y}$ | Common index: 15. |
| | Multiply first index and exponents by 3, second by 5 |
| $\sqrt[15]{x^9y^{12}x^{10}y^5}$ | Add exponents |
| $\sqrt[15]{x^{19}y^{17}}$ | Simplify by dividing exponents by index, remainder is left inside |
| $xy\,\sqrt[15]{x^4y^2}$ | Our Solution |

Just as with reducing the index, we will rewrite coefficients as exponential expressions. This will also allow us to use exponent properties to simplify.

**Example 7.**

| | |
|---|---|
| $\sqrt[3]{4x^2y}\,\sqrt[4]{8xy^3}$ | Rewrite 4 as $2^2$ and 8 as $2^3$ |
| $\sqrt[3]{2^2x^2y}\,\sqrt[4]{2^3xy^3}$ | Common index: 12. |
| | Multiply first index and exponents by 4, second by 3 |
| $\sqrt[12]{2^4x^8y^42^9x^3y^9}$ | Add exponents (even on the 2) |
| $\sqrt[12]{2^{13}x^{11}y^{13}}$ | Simplify by dividing exponents by index, remainder is left inside |
| $2y\,\sqrt[12]{2x^{11}y}$ | Our Solution |

If there is a binomial in the radical then we need to keep that binomial together through the entire problem.

**Example 8.**

| | |
|---|---|
| $\sqrt{3x(y+z)}\,\sqrt[3]{9x(y+z)^2}$ | Rewrite 9 as $3^2$ |
| $\sqrt{3x(y+z)}\,\sqrt[3]{3^2x(y+z)^2}$ | Common index: 6. Multiply first group by 3, second by 2 |
| $\sqrt[6]{3^3x^3(y+z)^3 3^4x^2(y+z)^4}$ | Add exponents, keep $(y+z)$ as binomial |
| $\sqrt[6]{3^7x^5(y+z)^7}$ | Simplify, dividing exponent by index, remainder inside |
| $3(y+z)\,\sqrt[6]{3x^5(y+z)}$ | Our Solution |

The same process is used for dividing mixed index as with multilpying mixed index. The only difference is our final answer can't have a radical over the denominator.

**Example 9.**

| | |
|---|---|
| $\dfrac{\sqrt[6]{x^4y^3z^2}}{\sqrt[8]{x^7y^2z}}$ | Common index is 24. Multiply first group by 4, second by 3 |
| $\sqrt[24]{\dfrac{x^{16}y^{12}z^8}{x^{21}y^6z^3}}$ | Subtract exponents |
| $\sqrt[24]{x^{-5}y^6z^5}$ | Negative exponent moves to denominator |
| $\sqrt[12]{\dfrac{y^6z^5}{x^5}}$ | Can$'t$ have denominator in radical, need $12x's$, or 7 more |
| $\sqrt[12]{\dfrac{y^6z^5}{x^5}}\left(\sqrt[12]{\dfrac{x^7}{x^7}}\right)$ | Multiply numerator and denominator by $\sqrt[12]{x^7}$ |
| $\dfrac{\sqrt[12]{x^7y^6z^5}}{x}$ | Our Solution |

# Practice - Radicals of Mixed Index

**Reduce the following radicals.**

1) $\sqrt[8]{16x^4y^6}$

2) $\sqrt[4]{9x^2y^6}$

3) $\sqrt[12]{64x^4y^6z^8}$

4) $\sqrt[4]{\frac{25x^3}{16x^5}}$

5) $\sqrt[6]{\frac{16x^2}{9y^4}}$

6) $\sqrt[15]{x^9y^{12}z^6}$

7) $\sqrt[12]{x^6y^9}$

8) $\sqrt[10]{64x^8y^4}$

9) $\sqrt[8]{8x^3y^6}$

10) $\sqrt[4]{25y^2}$

11) $\sqrt[9]{8x^3y^6}$

12) $\sqrt[16]{81x^8y^{12}}$

**Combine the following radicals.**

13) $\sqrt[3]{5}\sqrt{6}$

15) $\sqrt{x}\sqrt[3]{7y}$

17) $\sqrt{x}\sqrt[3]{x-2}$

19) $\sqrt[5]{x^2y}\sqrt{xy}$

21) $\sqrt[4]{xy^2}\sqrt[3]{x^2y}$

23) $\sqrt[4]{a^2bc^2}\sqrt[5]{a^2b^3c}$

25) $\sqrt{a}\sqrt[4]{a^3}$

27) $\sqrt[5]{b^2}\sqrt{b^3}$

29) $\sqrt{xy^3}\sqrt[3]{x^2y}$

31) $\sqrt[4]{9ab^3}\sqrt[4]{9x^3yz^2}$

33) $\sqrt[3]{3xy^2z}\sqrt[4]{9x^3yz^2}$

35) $\sqrt{27a^5(b+1)}\sqrt[3]{81a(b+1)^4}$

37) $\frac{\sqrt[3]{a^2}}{\sqrt[4]{a}}$

39) $\frac{\sqrt[4]{x^2y^3}}{\sqrt[3]{xy}}$

41) $\frac{\sqrt{ab^3c}}{\sqrt[5]{a^2b^3c^{-1}}}$

43) $\frac{\sqrt[4]{(3x-1)^3}}{\sqrt[5]{(3x-1)^3}}$

45) $\frac{\sqrt[3]{(2x+1)^2}}{\sqrt[5]{(2x+1)^2}}$

14) $\sqrt[3]{7}\sqrt[4]{5}$

16) $\sqrt[3]{y}\sqrt[5]{3z}$

18) $\sqrt[4]{3x}\sqrt{y+4}$

20) $\sqrt{ab}\sqrt[5]{2a^2b^2}$

22) $\sqrt[5]{a^2b^3}\ \sqrt[4]{a^2b}$

24) $\sqrt[6]{x^2yz^3}\ \sqrt[5]{x^2yz^2}$

26) $\sqrt[3]{x^2}\ \sqrt[6]{x^5}$

28) $\sqrt[4]{a^3}\ \sqrt[3]{a^2}$

30) $\sqrt[5]{a^3b}\ \sqrt{ab}$

32) $\sqrt{2x^3y^3}\ \sqrt[3]{4xy^2}$

34) $\sqrt{a^4b^3c^4}\ \sqrt[3]{ab^2c}$

36) $\sqrt{8x\,(y+z)^5}\ \sqrt[3]{4x^2(y+z)^2}$

38) $\frac{\sqrt[3]{x^2}}{\sqrt[5]{x}}$

40) $\frac{\sqrt[5]{a^4b^2}}{\sqrt[3]{ab^2}}$

42) $\frac{\sqrt[5]{x^3y^4z^9}}{\sqrt{xy^{-2}z}}$

44) $\frac{\sqrt[3]{(2+5x)^2}}{\sqrt[4]{(2+5x)}}$

46) $\frac{\sqrt[4]{(5-3x)^3}}{\sqrt[3]{(5-3x)^2}}$

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