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Rational Expressions - Add & Subtract

Objective: Add and subtract rational expressions with and without common denominators.

Adding and subtracting rational expressions is identical to adding and subtracting with integers. Recall that when adding with a common denominator we add the numerators and keep the denominator. This is the same process used with rational expressions. Remember to reduce, if possible, your final answer.

Example 1.

$$\frac{x-4}{x^2-2x-8}+\frac{x+8}{x^2-2x-8}$$ Same denominator, add numerators, combine like terms

$$\frac{2x+4}{x^2-2x-8}$$ Factor numerator and denominator

$$\frac{2(x+2)}{(x+2)(x-4)}$$ Divide out $(x+2)$

$$\frac{2}{x-4}$$ Our Solution

Subtraction with common denominator follows the same pattern, though the subtraction can cause problems if we are not careful with it. To avoid sign errors we will first distribute the subtraction through the numerator. Then we can treat it like an addition problem. This process is the same as "add the opposite" we saw when subtracting with negatives.

Example 2.

$$\frac{6x-12}{3x-6}-\frac{15x-6}{3x-6}$$ Add the opposite of the second fraction (distribute negative)

$$\frac{6x-12}{3x-6}+\frac{-15x+6}{3x-6}$$ Add numerators, combine like terms

$$\frac{-9x-6}{3x-6}$$ Factor numerator and denominator

$$\frac{-3(3x+2)}{3(x-2)}$$ Divide out common factor of 3

$$\frac{-(3x+2)}{x-2}$$ Our Solution

World View Note: The Rhind papyrus of Egypt from 1650 BC gives some of the earliest known symbols for addition and subtraction, a pair of legs walking in the direction one reads for addition, and a pair of legs walking in the opposite direction for subtraction..

When we don't have a common denominator we will have to find the least common denominator (LCD) and build up each fraction so the denominators match. The following example shows this process with integers.

Example 3.

$$\frac{5}{6}+\frac{1}{4}$$ The LCD is 12. Build up, multiply 6 by 2 and 4 by 3

$$\left(\frac{2}{2}\right)\frac{5}{6}+\frac{1}{4}\left(\frac{3}{3}\right)$$ Multiply

$$\frac{10}{12}+\frac{3}{12}$$ Add numerators

$$\frac{13}{12}$$ Our Solution

The same process is used with variables.

Example 4.

$$\frac{7a}{3a^2b}+\frac{4b}{6a\,b^4}$$ The LCD is $6a^2b^4$. We will then build up each fraction

$$\left(\frac{2b^3}{2b^3}\right)\frac{7a}{3a^2b}+\frac{4b}{6ab^4}\left(\frac{a}{a}\right)$$ Multiply first fraction by $2b^3$ and second by a

$$\frac{14ab^3}{6a^2b^4}+\frac{4ab}{6a^2b^4}$$ Add numerators, no like terms to combine

$$\frac{14ab^3+4ab}{6a^2b^4}$$ Factor numerator

$$\frac{2ab(7b^3+2)}{6a^2b^4}$$ Reduce, dividing out factors 2, a, and b

$$\frac{7b^3+2}{3ab^3}$$ Our Solution

The same process can be used for subtraction, we will simply add the first step of adding the opposite.

Example 5.

$$\frac{4}{5a}-\frac{7b}{4a^2}$$ Add the opposite

$$\frac{4}{5a}+\frac{-7b}{4a^2}$$ LCD is $20a^2$. Build up denominators

$$\left(\frac{4a}{4a}\right)\frac{4}{5a}+\frac{-7b}{4a^2}\left(\frac{5}{5}\right)$$ Multiply first fraction by $4a$, second by 5

$$\frac{16a-35b}{20a^2}$$ Our Solution

If our denominators have more than one term in them we will need to factor first to find the LCD. Then we build up each denominator using the factors that are missing on each fraction.

Example 6.

$$\frac{6}{8a+4}+\frac{3a}{8}$$ Factor denominators to find LCD

$$4(2a+1) \quad 8 \qquad \text{LCD is } 8(2a+1), \text{ build up each fraction}$$

$$\left(\frac{2}{2}\right)\frac{6}{4(2a+1)} + \frac{3a}{8}\left(\frac{2a+1}{2a+1}\right) \qquad \text{Multiply first fraction by 2, second by } 2a+1$$

$$\frac{12}{8(2a+1)} + \frac{6a^2+3a}{8(2a+1)} \qquad \text{Add numerators}$$

$$\frac{6a^2+3a+12}{8(2a+1)} \qquad \text{Our Solution}$$

With subtraction remember to add the opposite.

Example 7.

$$\frac{x+1}{x-4} - \frac{x+1}{x^2-7x+12} \qquad \text{Add the opposite (distribute negative)}$$

$$\frac{x+1}{x-4} + \frac{-x-1}{x^2-7x+12} \qquad \text{Factor denominators to find LCD}$$

$$x-4 \quad (x-4)(x-3) \qquad \text{LCD is } (x-4)(x-3), \text{ build up each fraction}$$

$$\left(\frac{x-3}{x-3}\right)\frac{x+1}{x-4} + \frac{-x-1}{x^2-7x+12} \qquad \text{Only first fraction needs to be multiplied by } x-3$$

$$\frac{x^2-2x-3}{(x-3)(x-4)} + \frac{-x-1}{(x-3)(x-4)} \qquad \text{Add numerators, combine like terms}$$

$$\frac{x^2-3x-4}{(x-3)(x-4)} \qquad \text{Factor numerator}$$

$$\frac{(x-4)(x+1)}{(x-3)(x-4)} \qquad \text{Divide out } x-4 \text{ factor}$$

$$\frac{x+1}{x-3} \qquad \text{Our Solution}$$

7.4 Practice - Add and Subtract

Add or subtract the rational expressions. Simplify your answers whenever possible.

1) $\frac{2}{a+3}+\frac{4}{a+3}$

2) $\frac{x^2}{x-2}-\frac{6x-8}{x-2}$

3) $\frac{t^2+4t}{t-1}+\frac{2t-7}{t-1}$

4) $\frac{a^2+3a}{a^2+5a-6}-\frac{4}{a^2+5a-6}$

5) $\frac{2x^2+3}{x^2-6x+5}-\frac{x^2-5x+9}{x^2-6x+5}$

6) $\frac{3}{x}+\frac{4}{x^2}$

7) $\frac{5}{6r}-\frac{5}{8r}$

8) $\frac{7}{xy^2}+\frac{3}{x^2y}$

9) $\frac{8}{9t^3}+\frac{5}{6t^2}$

10) $\frac{x+5}{8}+\frac{x-3}{12}$

11) $\frac{a+2}{2}-\frac{a-4}{4}$

12) $\frac{2a-1}{3a^2}+\frac{5a+1}{9a}$

13) $\frac{x-1}{4x}-\frac{2x+3}{x}$

14) $\frac{2c-d}{c^2d}-\frac{c+d}{cd^2}$

15) $\frac{5x+3y}{2x^2y}-\frac{3x+4y}{xy^2}$

16) $\frac{2}{x-1}+\frac{2}{x+1}$

17) $\frac{2z}{z-1}-\frac{3z}{z+1}$

18) $\frac{2}{x-5}+\frac{3}{4x}$

19) $\frac{8}{x^2-4}-\frac{3}{x+2}$

20) $\frac{4x}{x^2-25}+\frac{x}{x+5}$

21) $\frac{t}{t-3}-\frac{5}{4t-12}$

22) $\frac{2}{x+3}+\frac{4}{(x+3)^2}$

23) $\frac{2}{5x^2+5x}-\frac{4}{3x+3}$

24) $\frac{3a}{4a-20}+\frac{9a}{6a-30}$

25) $\frac{t}{y-t}-\frac{y}{y+t}$

26) $\frac{x}{x-5}+\frac{x-5}{x}$

27) $\frac{x}{x^2+5x+6}-\frac{2}{x^2+3x+2}$

28) $\frac{2x}{x^2-1}-\frac{3}{x^2+5x+4}$

29) $\frac{x}{x^2+15x+56}-\frac{7}{x^2+13x+42}$

30) $\frac{2x}{x^2-9}+\frac{5}{x^2+x-6}$

31) $\frac{5x}{x^2-x-6}-\frac{18}{x^2-9}$

32) $\frac{4x}{x^2-2x-3}-\frac{3}{x^2-5x+6}$

33) $\frac{2x}{x^2-1}-\frac{4}{x^2+2x-3}$

34) $\frac{x-1}{x^2+3x+2}+\frac{x+5}{x^2+4x+3}$

35) $\frac{x+1}{x^2-2x-35}+\frac{x+6}{x^2+7x+10}$

36) $\frac{3x+2}{3x+6}+\frac{x}{4-x^2}$

37) $\frac{4-a^2}{a^2-9}-\frac{a-2}{3-a}$

38) $\frac{4y}{y^2-1}-\frac{2}{y}-\frac{2}{y+1}$

39) $\frac{2z}{1-2z}+\frac{3z}{2z+1}-\frac{3}{4z^2-1}$

40) $\frac{2r}{r^2-s^2}+\frac{1}{r+s}-\frac{1}{r-s}$

41) $\frac{2x-3}{x^2+3x+2}+\frac{3x-1}{x^2+5x+6}$

42) $\frac{x+2}{x^2-4x+3}+\frac{4x+5}{x^2+4x-5}$

43) $\frac{2x+7}{x^2-2x-3}-\frac{3x-2}{x^2+6x+5}$

44) $\frac{3x-8}{x^2+6x+8}+\frac{2x-3}{x^2+3x+2}$

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Answers - Add and Subtract

1) $\frac{6}{a+3}$

2) $x-4$

3) $t+7$

4) $\frac{a+4}{a+6}$

5) $\frac{x+6}{x-5}$

6) $\frac{3x+4}{x^2}$

7) $\frac{5}{24r}$

8) $\frac{7x+3y}{x^2y^2}$

9) $\frac{15t+16}{18t^3}$

10) $\frac{5x+9}{24}$

11) $\frac{a+8}{4}$

12) $\frac{5a^2+7a-3}{9a^2}$

13) $\frac{-7x-13}{4x}$

14) $\frac{-c^2+cd-d^2}{c^2d^2}$

15) $\frac{3y^2-3xy-6x^2}{2x^2y^2}$

16) $\frac{4x}{x^2-1}$

17) $\frac{-z^2+5z}{z^2-1}$

18) $\frac{11x+15}{4x(x+5)}$

19) $\frac{14-3x}{x^2-4}$

20) $\frac{x^2-x}{x^2-25}$

21) $\frac{4t-5}{4(t-3)}$

22) $\frac{2x+10}{(x+3)^2}$

23) $\frac{6-20x}{15x(x+1)}$

24) $\frac{9a}{4(a-5)}$

25) $\frac{t^2+2ty-y^2}{y^2-t^2}$

26) $\frac{2x^2-10x+25}{x(x-5)}$

27) $\frac{x-3}{(x+3)(x+1)}$

28) $\frac{2x+3}{(x-1)(x+4)}$

29) $\frac{x-8}{(x+8)(x+6)}$

30) $\frac{2x-5}{(x-3)(x-2)}$

31) $\frac{5x+12}{x^2+5x+6}$

32) $\frac{4x+1}{(x+1)(x-2)}$

33) $\frac{2x+4}{x^2+4x+3}$

34) $\frac{2x+7}{x^2+5x+6}$

35) $\frac{2x-8}{x^2-5x-14}$

36) $\frac{-3x^2+7x+4}{3(x+2)(2-x)}$

37) $\frac{a-2}{a^2-9}$

38) $\frac{2}{y^2-y}$

39) $\frac{z-3}{2z-1}$

40) $\frac{2}{r+s}$

41) $\frac{5(x-1)}{(x+1)(x+3)}$

42) $\frac{5x+5}{x^2+2x-15}$

43) $\frac{-(x-29)}{(x-3)(x+5)}$

44) $\frac{5x-10}{x^2+5x+4}$