7.3

Rational Expressions - Least Common Denominators

Objective: Idenfity the least common denominator and build up denominators to match this common denominator.

As with fractions, the least common denominator or LCD is very important to working with rational expressions. The process we use to find the LCD is based on the process used to find the LCD of intergers.

Example 1.

Find the LCD of 8 and 6	Consider multiples of the larger number
$8, 16, 24 \ldots$	24 is the first multiple of 8 that is also divisible by 6
24	Our Solution

When finding the LCD of several monomials we first find the LCD of the coefficients, then use all variables and attach the highest exponent on each variable.

Example 2.

Find the LCD of $4x^2y^5$ and $6x^4y^3z^6$

	First find the LCD of coefficients 4 and 6
12	12 is the LCD of 4 and 6
$x^4y^5z^6$	Use all variables with highest exponents on each variable
$12x^4y^5z^6$	Our Solution

The same pattern can be used on polynomials that have more than one term. However, we must first factor each polynomial so we can identify all the factors to be used (attaching highest exponent if necessary).

Example 3.

Find the LCD of x^2+2x-3 and x^2-x-12	Factor each polynomial
$(x-1)(x+3)$ and $(x-4)(x+3)$	LCD uses all unique factors
$(x-1)(x+3)(x-4)$	Our Solution

Notice we only used $(x+3)$ once in our LCD. This is because it only appears as a factor once in either polynomial. The only time we need to repeat a factor or use an exponent on a factor is if there are exponents when one of the polynomials is factored

Example 4.

Find the LCD of $x^2-10x+25$ and $x^2-14x+45$

	Factor each polynomial
$(x-5)^2$ and $(x-5)(x-9)$	LCD uses all unique factors with highest exponent
$(x-5)^2(x-9)$	Our Solution

The previous example could have also been done with factoring the first polynomial to $(x-5)(x-5)$. Then we would have used $(x-5)$ twice in the LCD because it showed up twice in one of the polynomials. However, it is the author's suggestion to use the exponents in factored form so as to use the same pattern (highest exponent) as used with monomials.

Once we know the LCD, our goal will be to build up fractions so they have matching denominators. In this lesson we will not be adding and subtracting fractions, just building them up to a common denominator. We can build up a fraction's denominator by multipliplying the numerator and denoinator by any factors that are not already in the denominator.

Example 5.

$\dfrac{5a}{3a^2b}=\dfrac{?}{6a^5b^3}$	Idenfity what factors we need to match denominators
$2a^3b^2$	$3\cdot 2=6$ and we need three more $a's$ and two more $b's$
$\dfrac{5a}{3a^2b}\left(\dfrac{2a^3b^2}{2a^3b^2}\right)$	Multiply numerator and denominator by this
$\dfrac{10a^4b^2}{6a^5b^3}$	Our Solution

Example 6.

$\dfrac{x-2}{x+4}=\dfrac{?}{x^2+7x+12}$	Factor to idenfity factors we need to match denominators
$(x+4)(x+3)$	
$(x+3)$	The missing factor
$\dfrac{x-2}{x+4}\left(\dfrac{x+3}{x+3}\right)$	Multiply numerator and denominator by missing factor, FOIL numerator
$\dfrac{x^2+x-6}{(x+4)(x+3)}$	Our Solution

As the above example illustrates, we will multiply out our numerators, but keep our denominators factored. The reason for this is to add and subtract fractions we will want to be able to combine like terms in the numerator, then when we reduce at the end we will want our denominators factored.

Once we know how to find the LCD and how to build up fractions to a desired denominator we can combine them together by finding a common denominator and building up those fractions.

Example 7.

Build up each fraction so they have a common denominator

$$\frac{5a}{4b^3c} \text{ and } \frac{3c}{6a^2b} \quad \text{First identify LCD}$$

$$12a^2b^3c \quad \text{Determine what factors each fraction is missing}$$

$$\text{First: } 3a^2 \quad \text{Second: } 2b^2c \quad \text{Multiply each fraction by missing factors}$$

$$\frac{5a}{4b^3c}\left(\frac{3a^2}{3a^2}\right) \text{ and } \frac{3c}{6a^2b}\left(\frac{2b^2c}{2b^2c}\right)$$

$$\frac{15a^3}{12a^2b^3c} \text{ and } \frac{6b^2c^2}{12a^2b^3c} \quad \text{Our Solution}$$

Example 8.

Build up each fraction so they have a common denominator

$$\frac{5x}{x^2-5x-6} \text{ and } \frac{x-2}{x^2+4x+3} \quad \text{Factor to find LCD}$$

$$(x-6)(x+1) \qquad (x+1)(x+3) \quad \text{Use factors to find LCD}$$

$$\text{LCD: } (x-6)(x+1)(x+3) \quad \text{Identify which factors are missing}$$

$$\text{First: } (x+3) \quad \text{Second: } (x-6) \quad \text{Multiply fractions by missing factors}$$

$$\frac{5x}{(x-6)(x+1)}\left(\frac{x+3}{x+3}\right) \text{ and } \frac{x-2}{(x+1)(x+3)}\left(\frac{x-6}{x-6}\right) \quad \text{Multiply numerators}$$

$$\frac{5x^2+15x}{(x-6)(x+1)(x+3)} \text{ and } \frac{x^2-8x+12}{(x-6)(x+1)(x+3)} \quad \text{Our Solution}$$

World View Note: When the Egyptians began working with fractions, they expressed all fractions as a sum of unit fraction. Rather than $\frac{4}{5}$, they would write the fraction as the sum, $\frac{1}{2}+\frac{1}{4}+\frac{1}{20}$. An interesting problem with this system is this is not a unique solution, $\frac{4}{5}$ is also equal to the sum $\frac{1}{3}+\frac{1}{5}+\frac{1}{6}+\frac{1}{10}$.

7.3 Practice - Least Common Denominator

Build up denominators.

1) $\frac{3}{8} = \frac{?}{48}$

2) $\frac{a}{5} = \frac{?}{5a}$

3) $\frac{a}{x} = \frac{?}{xy}$

4) $\frac{5}{2x^2} = \frac{?}{8x^3y}$

5) $\frac{2}{3a^3b^2c} = \frac{?}{9a^5b^2c^4}$

6) $\frac{4}{3a^5b^2c^4} = \frac{?}{9a^5b^2c^4}$

7) $\frac{2}{x+4} = \frac{?}{x^2-16}$

8) $\frac{x+1}{x-3} = \frac{?}{x^2-6x+9}$

9) $\frac{x-4}{x+2} = \frac{?}{x^2+5x+6}$

10) $\frac{x-6}{x+3} = \frac{?}{x^2-2x-15}$

Find Least Common Denominators

11) $2a^3, 6a^4b^2, 4a^3b^5$

12) $5x^2y, 25x^3y^5z$

13) $x^2-3x, x-3, x$

14) $4x-8, x-2, 4$

15) $x+2, x-4$

16) $x, x-7, x+1$

17) $x^2-25, x+5$

18) x^2-9, x^2-6x+9

19) x^2+3x+2, x^2+5x+6

20) $x^2-7x+10, x^2-2x-15, x^2+x-6$

Find LCD and build up each fraction

21) $\frac{3a}{5b^2}, \frac{2}{10a^3b}$

22) $\frac{3x}{x-4}, \frac{2}{x+2}$

23) $\frac{x+2}{x-3}, \frac{x-3}{x+2}$

24) $\frac{5}{x^2-6x}, \frac{2}{x}, \frac{-3}{x-6}$

25) $\frac{x}{x^2-16}, \frac{3x}{x^2-8x+16}$

26) $\frac{5x+1}{x^2-3x-10}, \frac{4}{x-5}$

27) $\frac{x+1}{x^2-36}, \frac{2x+3}{x^2+12x+36}$

28) $\frac{3x+1}{x^2-x-12}, \frac{2x}{x^2+4x+3}$

29) $\frac{4x}{x^2-x-6}, \frac{x+2}{x-3}$

30) $\frac{3x}{x^2-6x+8}, \frac{x-2}{x^2+x-20}, \frac{5}{x^2+3x-10}$

Answers - Least Common Denominators

1) 18

2) a^2

3) ay

4) $20xy$

5) $6a^2c^3$

6) 12

7) $2x-8$

8) x^2-2x-3

9) x^2-x-12

10) $x^2-11x+30$

11) $12a^4b^5$

12) $25x^3y^5z$

13) $x(x-3)$

14) $4(x-2)$

15) $(x+2)(x-4)$

16) $x(x-7)(x+1)$

17) $(x+5)(x-5)$

18) $(x-3)^2(x+3)$

19) $(x+1)(x+2)(x+3)$

20) $(x-2)(x-5)(x+3)$

21) $\frac{6a^4}{10a^3b^2}, \frac{2b}{10a^3b^2}$

22) $\frac{3x^2+6x}{(x-4)(x+2)}, \frac{2x-8}{(x-4)(x+2)}$

23) $\frac{x^2+4x+4}{(x-3)(x+2)}, \frac{x^2-6x+9}{(x-3)(x+2)}$

24) $\frac{5}{x(x-6)}, \frac{2x-12}{x(x-6)}, \frac{-3x}{x(x-6)}$

25) $\frac{x^2-4x}{(x-4)^2(x+4)}, \frac{3x^2+12x}{(x-4)^2(x+4)}$

26) $\frac{5x+1}{(x-5)(x+2)}, \frac{4x+8}{(x-5)(x+2)}$

27) $\frac{x^2+7x+6}{(x-6)(x+6)^2}, \frac{2x^2-9x-18}{(x-6)(x+6)^2}$

28) $\frac{3x^2+4x+1}{(x-4)(x+3)(x+1)}, \frac{2x^2-8x}{(x-4)(x+3)(x+1)}$

29) $\frac{4x}{(x-3)(x+2)}, \frac{x^2+4x+4}{(x-3)(x+2)}$

30) $\frac{3x^2+15x}{(x-4)(x-2)(x+5)}, \frac{x^2-4x+4}{(x-4)(x-2)(x+5)}, \frac{5x-20}{(x-4)(x-2)(x+5)}$