# Rational Expressions - Reduce Rational Expressions

**Objective: Reduce rational expressions by dividing out common factors.**

**Rational expressions** are expressions written as a quotient of polynomials. Examples of rational expressions include:

$$\frac{x^2-x-12}{x^2-9x+20} \quad \text{and} \quad \frac{3}{x-2} \quad \text{and} \quad \frac{a-b}{b-a} \quad \text{and} \quad \frac{3}{2}$$

As rational expressions are a special type of fraction, it is important to remember with fractions we cannot have zero in the denominator of a fraction. For this reason, rational expressions may have one more excluded values, or values that the variable cannot be or the expression would be undefined.

**Example 1.**

$$\begin{array}{rl}
\text{State the excluded value}(s)\text{:}\ \dfrac{x^2-1}{3x^2+5x} & \text{Denominator cannot be zero} \\
3x^2+5x\neq 0 & \text{Factor} \\
x(3x+5)\neq 0 & \text{Set each factor not equal to zero} \\
x\neq 0 \ \text{ or } \ 3x+5\neq 0 & \text{Subtract 5 from second equation} \\
\underline{-5\,-5} & \\
3x\neq -5 & \text{Divide by 3} \\
\overline{3}\quad\ \overline{3} & \\
x\neq \dfrac{-5}{3} & \text{Second equation is solved} \\
x\neq 0 \ \text{ or } \ \dfrac{-5}{3} & \text{Our Solution}
\end{array}$$

This means we can use any value for $x$ in the equation except for 0 and $\frac{-5}{3}$. We can however, evaluate any other value in the expression.

**World View Note:** The number zero was not widely accepted in mathematical thought around the world for many years. It was the Mayans of Central America who first used zero to aid in the use of their base 20 system as a place holder!

Rational expressions are easily evaluated by simply substituting the value for the variable and using order of operations.

**Example 2.**

$$\frac{x^2-4}{x^2+6x+8} \text{ when } x=-6 \quad \text{Substitute} -5 \text{ in for each variable}$$

$$\frac{(-6)^2-4}{(-6)^2+6(-6)+8} \quad \text{Exponents first}$$

$$\frac{36-4}{36+6(-6)+8} \quad \text{Multiply}$$

$$\frac{36-4}{36-36+8} \quad \text{Add and subtract}$$

$$\frac{32}{8} \quad \text{Reduce}$$

$$4 \quad \text{Our Solution}$$

Just as we reduced the previous example, often a rational expression can be reduced, even without knowing the value of the variable. When we reduce we divide out common factors. We have already seen this with monomials when we discussed properties of exponents. If the problem only has monomials we can reduce the coefficients, and subtract exponents on the variables.

**Example 3.**

$$\frac{15x^4y^2}{25x^2y^6} \quad \text{Reduce , subtract exponents. Negative exponents move to denominator}$$

$$\frac{3x^2}{5y^4} \quad \text{Our Solution}$$

However, if there is more than just one term in either the numerator or denominator, we can't divide out common factors unless we first factor the numerator and denominator.

**Example 4.**

$$\frac{28}{8x^2-16} \quad \text{Denominator has } a \text{ common factor of } 8$$

$$\frac{28}{8(x^2-2)} \quad \text{Reduce by dividing 24 and 8 by 4}$$

$$\frac{7}{2(x^2-2)} \quad \text{Our Solution}$$

**Example 5.**

$$\frac{9x-3}{18x-6} \quad \text{Numerator has } a \text{ common factor of } 3\text{, denominator of } 6$$

$$\frac{3(3x-1)}{6(3x-1)} \quad \text{Divide out common factor } (3x-1) \text{ and divide 3 and 6 by 3}$$

$$\frac{1}{2} \quad \text{Our Solution}$$

**Example 6.**

$$\frac{x^2-25}{x^2+8x+15} \quad \text{Numerator is difference of squares, denominator is factored using ac}$$

$$\frac{(x+5)(x-5)}{(x+3)(x+5)} \quad \text{Divide out common factor } (x+5)$$

$$\frac{x-5}{x+3} \quad \text{Our Solution}$$

It is important to remember we cannot reduce terms, only factors. This means if there are any $+$ or $-$ between the parts we want to reduce we cannot. In the previous example we had the solution $\frac{x-5}{x+3}$, we cannot divide out the $x$'s because they are terms (separated by $+$ or $-$) not factors (separated by multiplication).

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# 7.1 Practice - Reduce Rational Expressions

**Evaluate**

1) $\frac{4v+2}{6}$ when $v=4$

2) $\frac{b-3}{3b-9}$ when $b=-2$

3) $\frac{x-3}{x^2-4x+3}$ when $x=-4$

4) $\frac{a+2}{a^2+3a+2}$ when $a=-1$

5) $\frac{b+2}{b^2+4b+4}$ when $b=0$

6) $\frac{n^2-n-6}{n-3}$ when $n=4$

**State the excluded values for each.**

7) $\frac{3k^2+30k}{k+10}$

8) $\frac{27p}{18p^2-36p}$

9) $\frac{15n^2}{10n+25}$

10) $\frac{x+10}{8x^2+80x}$

11) $\frac{10m^2+8m}{10m}$

12) $\frac{10x+16}{6x+20}$

13) $\frac{r^2+3r+2}{5r+10}$

14) $\frac{6n^2-21n}{6n^2+3n}$

15) $\frac{b^2+12b+32}{b^2+4b-32}$

16) $\frac{10v^2+30v}{35v^2-5v}$

**Simplify each expression.**

17) $\frac{21x^2}{18x}$

18) $\frac{12n}{4n^2}$

19) $\frac{24a}{40a^2}$

20) $\frac{21k}{24k^2}$

21) $\frac{32x^3}{8x^4}$

22) $\frac{90x^2}{20x}$

23) $\frac{18m-24}{60}$

24) $\frac{10}{81n^3+36n^2}$

25) $\frac{20}{4p+2}$

26) $\frac{n-9}{9n-81}$

27) $\frac{x+1}{x^2+8x+7}$

28) $\frac{28m+12}{36}$

29) $\frac{32x^2}{28x^2+28x}$

30) $\frac{49r+56}{56r}$

31) $\frac{n^2+4n-12}{n^2-7n+10}$

32) $\frac{b^2+14b+48}{b^2+15b+56}$

33) $\frac{9v+54}{v^2-4v-60}$

34) $\frac{30x-90}{50x+40}$

35) $\frac{12x^2-42x}{30x^2-42x}$

36) $\frac{k^2-12k+32}{k^2-64}$

37) $\frac{6a-10}{10a+4}$

38) $\frac{9p+18}{p^2+4p+4}$

39) $\frac{2n^2+19n-10}{9n+90}$

40) $\frac{3x^2-29x+40}{5x^2-30x-80}$

41) $\frac{8m+16}{20m-12}$

42) $\frac{56x-48}{24x^2+56x+32}$

43) $\frac{2x^2-10x+8}{3x^2-7x+4}$

44) $\frac{50b-80}{50b+20}$

45) $\frac{7n^2-32n+16}{4n-16}$

46) $\frac{35v+35}{21v+7}$

47) $\frac{n^2-2n+1}{6n+6}$

48) $\frac{56x-48}{24x^2+56x+32}$

49) $\frac{7a^2-26a-45}{6a^2-34a+20}$

50) $\frac{4k^3-2k^2-2k}{9k^3-18k^2+9k}$

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# Answers - Reduce Rational Expressions

1) $3$

2) $\frac{1}{3}$

3) $-\frac{1}{5}$

4) undefined

5) $\frac{1}{2}$

6) $6$

7) $-10$

8) $0, 2$

9) $-\frac{5}{2}$

10) $0, -10$

11) $0$

12) $-\frac{10}{3}$

13) $-2$

14) $0, -\frac{1}{2}$

15) $-8, 4$

16) $0, \frac{1}{7}$

17) $\frac{7x}{6}$

18) $\frac{3}{n}$

19) $\frac{3}{5a}$

20) $\frac{7}{8k}$

21) $\frac{4}{x}$

22) $\frac{9x}{2}$

23) $\frac{3m-4}{10}$

24) $\frac{10}{9n^2(9n+4)}$

25) $\frac{10}{2p+1}$

26) $\frac{1}{9}$

27) $\frac{1}{x+7}$

28) $\frac{7m+3}{9}$

29) $\frac{8x}{7(x+1)}$

30) $\frac{7r+8}{8r}$

31) $\frac{n+6}{n-5}$

32) $\frac{b+6}{b+7}$

33) $\frac{9}{v-10}$

34) $\frac{3(x-3)}{5x+4}$

35) $\frac{2x-7}{5x-7}$

36) $\frac{k-8}{k+4}$

37) $\frac{3a-5}{5a+2}$

38) $\frac{9}{p+2}$

39) $\frac{2n-1}{9}$

40) $\frac{3x-5}{5(x+2)}$

41) $\frac{2(m+2)}{5m-3}$

42) $\frac{9r}{5(r+1)}$

43) $\frac{2(x-4)}{3x-4}$

44) $\frac{5b-8}{5b+2}$

45) $\frac{7n-4}{4}$

46) $\frac{5(v+1)}{3v+1}$

47) $\frac{(n-1)^2}{6(n+1)}$

48) $\frac{7x-6}{(3x+4)(x+1)}$

49) $\frac{7a+9}{2(3a-2)}$

50) $\frac{2(2k+1)}{9(k-1)}$

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