Factoring - Factoring Strategy

Objective: Idenfity and use the correct method to factor various polynomials.

With so many different tools used to factor, it is easy to get lost as to which tool to use when. Here we will attempt to organize all the different factoring types we have seen. A large part of deciding how to solve a problem is based on how many terms are in the problem. For all problem types we will always try to factor out the GCF first.

Factoring Strategy (GCF First!!!!!)

- **2 terms:** sum or difference of squares or cubes:

  $a^2 - b^2 = (a+b)(a-b)$

  $a^2 + b^2 = \text{Prime}$

  $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

  $a^3 - b^3 = (a-b)(a^2 + \text{ab} + b^2)$

- **3 terms:** ac method, watch for perfect square!

  $a^2 + 2ab + b^2 = (a+b)^2$

  Multiply to ac and add to b

- **4 terms:** grouping

We will use the above strategy to factor each of the following examples. Here the emphasis will be on which strategy to use rather than the steps used in that method.

Example 1.

$$
\begin{array}{rl}
4x^2 + 56xy + 196y^2 & \text{GCF first, } 4 \\
4(x^2 + 14xy + 49y^2) & \text{Three terms, try ac method, multiply to } 49\text{, add to } 14 \\
 & 7 \text{ and } 7\text{, perfect square!} \\
4(x+7y)^2 & \text{Our Solution}
\end{array}
$$

Example 2.

$5x^2y+15xy-35x^2-105x$	GCF first, $5x$
$5x(xy+3y-7x-21)$	Four terms, try grouping
$5x[y(x+3)-7(x+3)]$	$(x+3)$ match!
$5x(x+3)(y-7)$	Our Solution

Example 3.

$100x^2-400$	GCF first, 100
$100(x^2-4)$	Two terms, difference of squares
$100(x+4)(x-4)$	Our Solution

Example 4.

$108x^3y^2-39x^2y^2+3xy^2$	GCF first, $3xy^2$
$3xy^2(36x^2-13x+1)$	Thee terms, ac method, multiply to 36, add to -13
$3xy^2(36x^2-9x-4x+1)$	-9 and -4, split middle term
$3xy^2[9x(4x-1)-1(4x-1)]$	Factor by grouping
$3xy^2(4x-1)(9x-1)$	Our Solution

World View Note: Variables originated in ancient Greece where Aristotle would use a single capital letter to represent a number.

Example 5.

$5+625y^3$	GCF first, 5
$5(1+125y^3)$	Two terms, sum of cubes
$5(1+5y)(1-5y+25y^2)$	Our Solution

It is important to be comfortable and confident not just with using all the factoring methods, but decided on which method to use. This is why practice is very important!

6.6 Practice - Factoring Strategy

Factor each completely.

1) $24az - 18ah + 60yz - 45yh$

2) $2x^2 - 11x + 15$

3) $5u^2 - 9uv + 4v^2$

4) $16x^2 + 48xy + 36y^2$

5) $-2x^3 + 128y^3$

6) $20uv - 60u^3 - 5xv + 15xu^2$

7) $5n^3 + 7n^2 - 6n$

8) $2x^3 + 5x^2y + 3y^2x$

9) $54u^3 - 16$

10) $54 - 128x^3$

11) $n^2 - n$

12) $5x^2 - 22x - 15$

13) $x^2 - 4xy + 3y^2$

14) $45u^2 - 150uv + 125v^2$

15) $9x^2 - 25y^2$

16) $x^3 - 27y^3$

17) $m^2 - 4n^2$

18) $12ab - 18a + 6nb - 9n$

19) $36b^2c - 16xd - 24b^2d + 24xc$

20) $3m^3 - 6m^2n - 24n^2m$

21) $128 + 54x^3$

22) $64m^3 + 27n^3$

23) $2x^3 + 6x^2y - 20y^2x$

24) $3ac + 15ad^2 + x^2c + 5x^2d^2$

25) $n^3 + 7n^2 + 10n$

26) $64m^3 - n^3$

27) $27x^3 - 64$

28) $16a^2 - 9b^2$

29) $5x^2 + 2x$

30) $2x^2 - 10x + 12$

31) $3k^3 - 27k^2 + 60k$

32) $32x^2 - 18y^2$

33) $mn - 12x + 3m - 4xn$

34) $2k^2 + k - 10$

35) $16x^2 - 8\text{xy} + y^2$

36) $v^2 + v$

37) $27m^2 - 48n^2$

38) $x^3 + 4x^2$

39) $9x^3 + 21x^2y - 60y^2x$

40) $9n^3 - 3n^2$

41) $2m^2 + 6mn - 20n^2$

42) $2u^2v^2 - 11uv^3 + 15v^4$

Answers - Factoring Strategy

1) $3(2a+5y)(4z-3h)$
2) $(2x-5)(x-3)$
3) $(5u-4v)(u-v)$
4) $4(2x+3y)^2$
5) $2(-x+4y)(x^2+4xy+16y^2)$
6) $5(4u-x)(v-3u^2)$
7) $n(5n-3)(n+2)$
8) $x(2x+3y)(x+y)$
9) $2(3u-2)(9u^2+6u+4)$
10) $2(3-4x)(9+12x+16x^2)$
11) $n(n-1)$
12) $(5x+3)(x-5)$
13) $(x-3y)(x-y)$
14) $5(3u-5v)^2$
15) $(3x+5y)(3x-5y)$
16) $(x-3y)(x^2+3xy+9y^2)$
17) $(m+2n)(m-2n)$
18) $3(2a+n)(2b-3)$
19) $4(3b^2+2x)(3c-2d)$
20) $3m(m+2n)(m-4n)$
21) $2(4+3x)(16-12x+9x^2)$
22) $(4m+3n)(16m^2-12mn+9n^2)$
23) $2x(x+5y)(x-2y)$
24) $(3a+x^2)(c+5d^2)$
25) $n(n+2)(n+5)$
26) $(4m-n)(16m^2+4mn+n^2)$
27) $(3x-4)(9x^2+12x+16)$
28) $(4a+3b)(4a-3b)$
29) $x(5x+2)$
30) $2(x-2)(x-3)$
31) $3k(k-5)(k-4)$
32) $2(4x+3y)(4x-3y)$
33) $(m-4x)(n+3)$
34) $(2k+5)(k-2)$
35) $(4x-y)^2$
36) $v(v+1)$
37) $3(3m+4n)(3m-4n)$
38) $x^2(x+4)$
39) $3x(3x-5y)(x+4y)$
40) $3n^2(3n-1)$
41) $2(m-2n)(m+5n)$
42) $v^2(2u-5v)(u-3v)$