# Functions - Inverse Trigonometry

We used a special function, one of the trig functions, to take an angle of a triangle and find the side length. Here we will do the opposite, take the side lengths and find the angle. Because this is the opposite operation, we will use the inverse function of each of the trig ratios we saw before. The notation we will use for the inverse trig functions will be similar to the inverse notation we used with functions.

$$\mathbf{sin^{-1}}\left(\frac{\textbf{opposite}}{\textbf{hypotenuse}}\right)=\boldsymbol{\theta} \quad \mathbf{cos^{-1}}\left(\frac{\textbf{adjacent}}{\textbf{hypotenuse}}\right)=\boldsymbol{\theta} \quad \mathbf{tan^{-1}}\left(\frac{\textbf{opposite}}{\textbf{adjacent}}\right)=\boldsymbol{\theta}$$

Just as with inverse functions, the $-1$ is not an exponent, it is a notation to tell us that these are inverse functions. While the regular trig functions take angles as inputs, these inverse functions will always take a ratio of sides as inputs. We can calculate inverse trig values using a table or a calculator (usually pressing shift or 2nd first).

**Example 1.**

$$\begin{aligned} \sin\mathrm{A} &= 0.5 && \text{We don't know the angle so we use an inverse trig function} \\ \sin^{-1}(0.5) &= A && \text{Evaluate using table or calculator} \\ 30^\circ &= A && \text{Our Solution} \end{aligned}$$

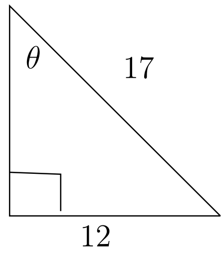
$$\begin{aligned} \cos\mathrm{B} &= 0.667 && \text{We don't know the angle so we use an inverse trig function} \\ \cos^{-1}(0.667) &= B && \text{Evaluate using table or calculator} \\ 48^\circ &= B && \text{Our Solution} \end{aligned}$$

$$\begin{aligned} \tan\mathrm{C} &= 1.54 && \text{We don't know the angle so we use an inverse trig function} \\ \tan^{-1}(1.54) &= C && \text{Evaluate using table or calculator} \\ 57^\circ &= C && \text{Our Solution} \end{aligned}$$

If we have two sides of a triangle, we can easily calculate their ratio as a decimal and then use one of the inverse trig functions to find a missing angle.

**Example 2.**

Find the indicated angle.




From angle $\theta$ the given sides are the opposite (12) and the hypotenuse (17).

The trig functions that uses opposite and hypotenuse is the sine

Because we are lookin for an angle we use the inverse sine

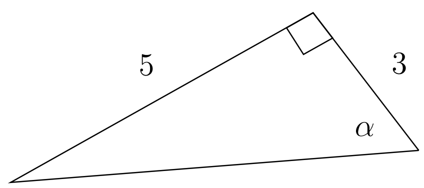
$$\sin^{-1}\left(\frac{12}{17}\right) \quad \text{Sine is opposite over hyptenuse, use inverse to find angle}$$

$$\sin^{-1}(0.706) \quad \text{Evaluate fraction, take sine inverse using table or calculator}$$

$$45^\circ \quad \text{Our Solution}$$

**Example 3.**

Find the indicated angle




From the angle $\alpha$, the given sides are the opposite (5) and the adjacent (3)

The trig function that uses opposite and adjacent is the tangent

As we are lookin for an angle we ill use the inverse tangent.

$$\tan^{-1}\left(\frac{5}{3}\right) \quad \text{Tangent is opposite over adjacent. Use inverse to find angle}$$

$$\tan^{-1}(1.667) \quad \text{Evaluate fraction, take tangent inverse on table or calculator}$$

$$50^\circ \quad \text{Our Solution}$$

Using a combination of trig functions and inverse trig functions, if we are given two parts of a right triangle (two sides or a side and an angle), we can find all the other sides and angles of the triangle. This is called solving a triangle.

When we are solving a triangle, we can use trig ratios to solve for all the missing parts of it, but there are some properties from geometry that may be helpful along the way.

The angles of a triangle always add up to $180^\circ$, because we have a right triangle, $90^\circ$ are used up in the right angle, that means there are another $90^\circ$ left in the two acute angles. In other words, the smaller two angles will always add to 90, if we know one angle, we can quickly find the other by subtracting from 90.

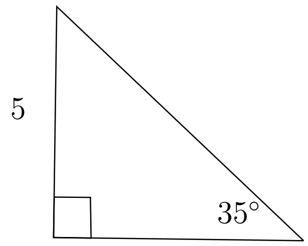
Another trick is on the sides of the angles. If we know two sides of the right triangle, we can use the Pythagorean Theorem to find the third side. The Pythagorean Theorem states that if $c$ is the hypotenuse of the triangle, and $a$ and

$b$ are the other two sides (legs), then we can use the following formula, $a^2+b^2=c^2$ to find a missing side.

Often when solving triangles we use trigonometry to find one part, then use the angle sum and/or the Pythagorean Theorem to find the other two parts.

**Example 4.**

Solve the triangle




We have one angle and one side. We can use these to find either other side. We will find the other leg, the adjacent side to the 35° angle.

The 5 is the opposite side, so we will use the tangent to find the leg.

$$\tan 35^\circ = \frac{5}{x} \quad \text{Tangent is opposite over adjacent}$$

$$\frac{0.700}{1} = \frac{5}{x} \quad \text{Evaluate tangent, put it over one so we have } a \text{ proportion}$$

$$0.700x = 5 \quad \text{Find cross product}$$

$$\overline{0.700} \quad \overline{0.700} \quad \text{Divide both sides by 0.700}$$

$$x = 7.1 \quad \text{The missing leg.}$$

$$a^2+b^2=c^2 \quad \text{We can now use pythagorean thorem to find hypotenuse, } c$$

$$5^2+7.1^2=c^2 \quad \text{Evaluate exponents}$$

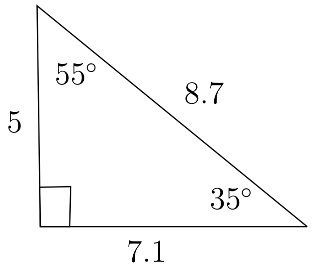
$$25+50.41=c^2 \quad \text{Add}$$

$$75.41=c^2 \quad \text{Square root both sides}$$

$$8.7=c \quad \text{The hypotenuse}$$

$$90^\circ-35^\circ \quad \text{To find the missing angle we subtract from } 90^\circ$$

$$55^\circ \quad \text{The missing angle}$$

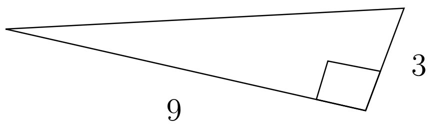



Our Solution

In the previous example, once we found the leg to be 7.1 we could have used the sine function on the 35° angle to get the hypotenuse and then any inverse trig function to find the missing angle and we would have found the same answers. The angle sum and pythagorean theorem are just nice shortcuts to solve the problem quicker.

**Example 5.**

Solve the triangle




In this triangle we have two sides. We will first find the angle on the right side, adjacent to 3 and opposite from the 9.

Tangent uses opposite and adjacent

To find an angle we use the inverse tangent.

$\tan^{-1}\left(\frac{9}{3}\right)$ Evaluate fraction

$\tan^{-1}(3)$ Evaluate tangent

$71.6^\circ$ The angle on the right side

$90^\circ - 71.6^\circ$ Subtract angle from 90 to get other angle

$18.4^\circ$ The angle on the left side

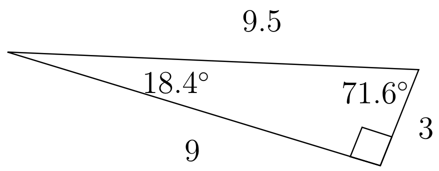
$a^2 + b^2 = c^2$ Pythagorean theorem to find hypotenuse

$9^2 + 3^2 = c^2$ Evaluate exponents

$81 + 9 = c^2$ Add

$90 = c^2$ Square root both sides

$3\sqrt{10}$ or $9.5 = c$ The hypotenuse




Our Solution

# Practice - Inverse Trigonometry

**Find each angle measure to the nearest degree.**

1) sin Z = 0.4848

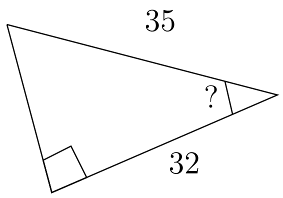
2) sin Y = 0.6293

3) sin Y = 0.6561

4) cos Y = 0.6157

**Find the measure of the indicated angle to the nearest degree.**

5)

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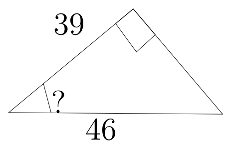


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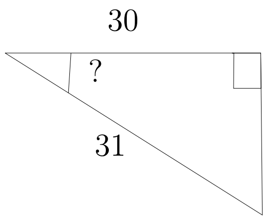


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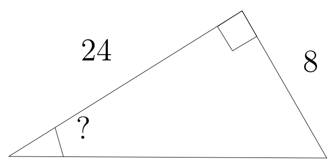


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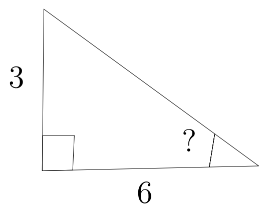


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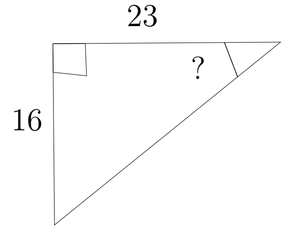

**Find the measure of each angle indicated. Round to the nearest tenth.**

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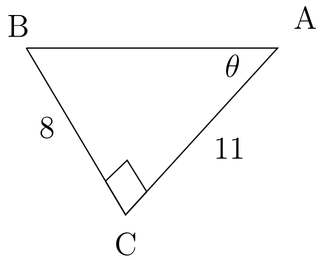


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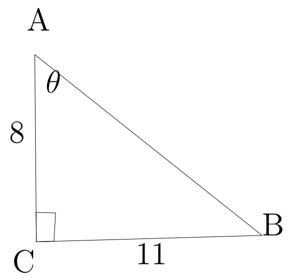


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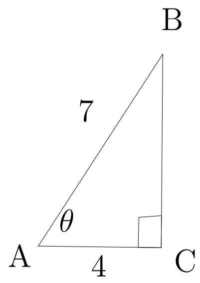


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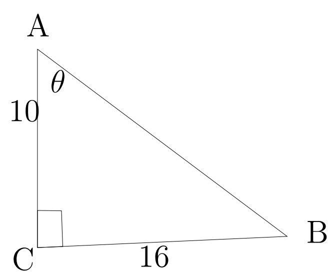


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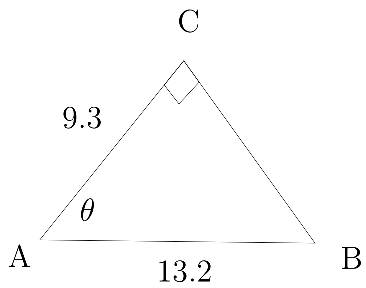


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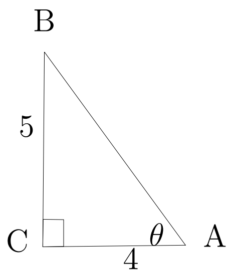


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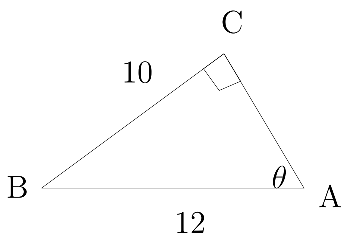


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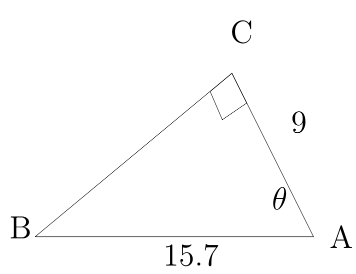


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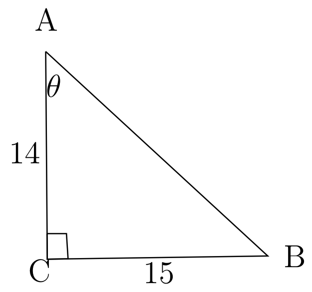


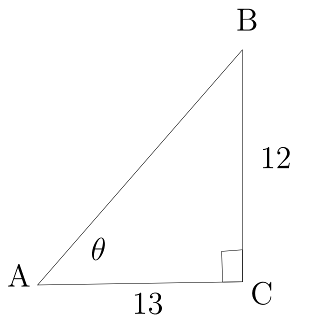


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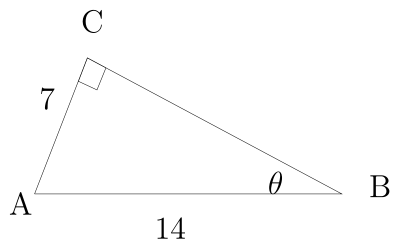


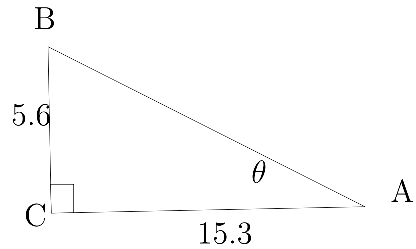


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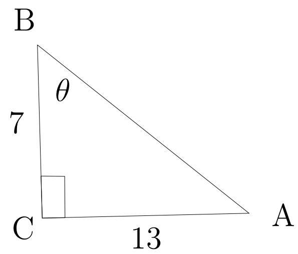


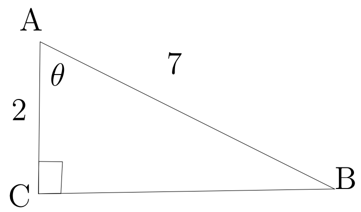


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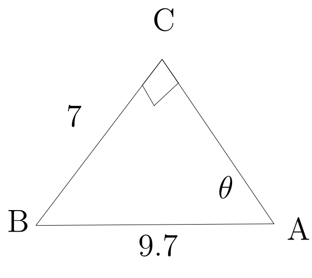


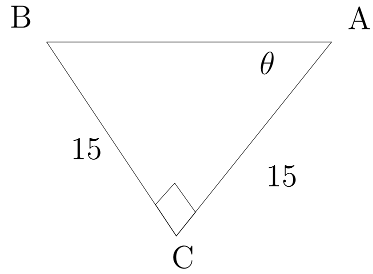


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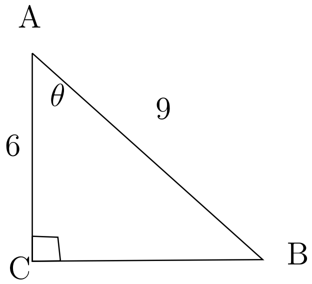


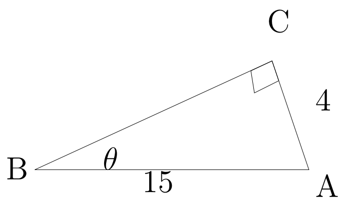


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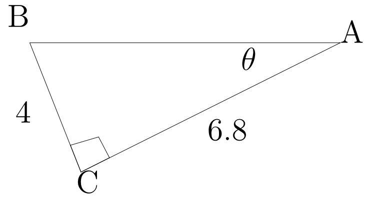
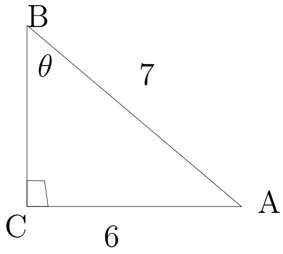
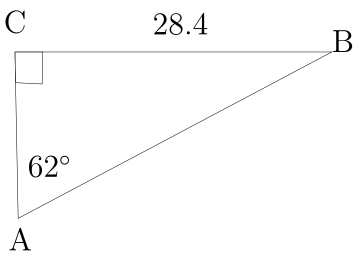


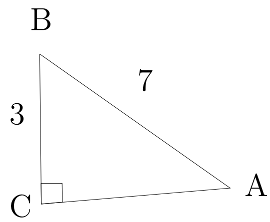




28)

**Solve each triangle. Round answers to the nearest tenth.**

31)

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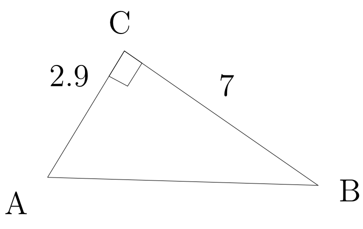


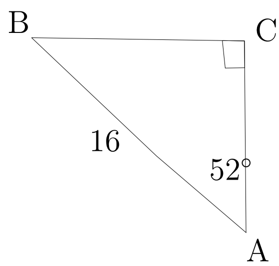


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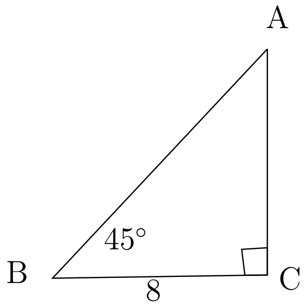


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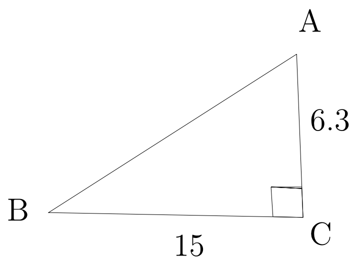


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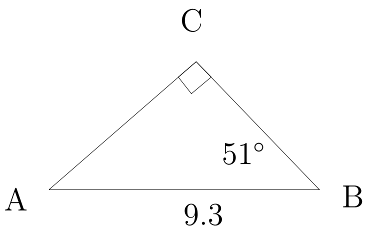


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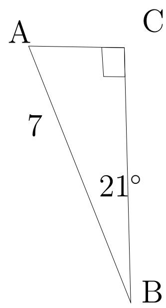


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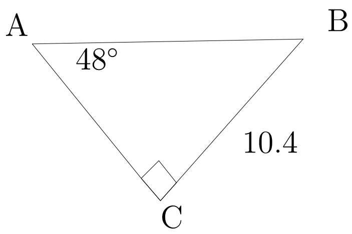


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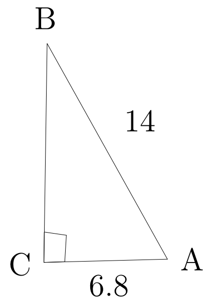

## Answers - Inverse Trigonometry

1) $29°$
2) $39°$
3) $41°$
4) $52°$
5) $24°$
6) $32°$
7) $15°$
8) $18°$
9) $27°$
10) $35°$
11) $36°$
12) $61.7°$
13) $54°$
14) $46.2°$
15) $55.2°$
16) $42.7°$
17) $58°$
18) $20.1°$
19) $45.2°$
20) $73.4°$
21) $51.3°$
22) $45°$
23) $56.4°$
24) $48.2°$
25) $55°$
26) $30.5°$
27) $47°$
28) $15.5°$
29) $30°$
30) $59°$
31) $m\angle B = 28°, b = 141, c = 32.2$
32) $m\angle B = 22.8°, m\angle A = 67.2°, c = 16.3$
33) $m\angle B = 22.5°, m\angle A = 67.5°, c = 7.6$
34) $m\angle A = 39°, b = 7.2, a = 5.9$
35) $m\angle B = 64.6°, m\angle A = 25.4°, b = 6.3$
36) $m\angle A = 69°, b = 2.5, a = 6.5$
37) $m\angle B = 38°, b = 9.9, a = 12.6$
38) $m\angle B = 42°, b = 9.4, c = 14$
39) $m\angle A = 45°, b = 8, c = 11.3$
40) $m\angle B = 29.1°, m\angle A = 60.9°, a = 12.2$