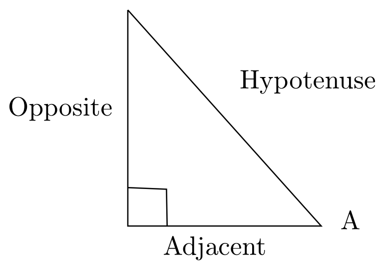
Functions - Trigonometry

There are six special functions that describe the relationship between the sides of a right triangle and the angles of the triangle. We will discuss three of the functions here. The three functions are called the sine, cosine, and tangent (the three others are cosecant, secant, and cotangent, but we will not need to use them here).

To the right is a picture of a right triangle. Based on which angle we are interested in on a given problem we will name the three sides in relationship to that angle. In the picture, angle A is the angle we will use to name the other sides. The longest side, the side opposite the right angle is always called the hypotenouse. The side across from the angle A is called the opposite side.



The third side, the side between our angle and the right angle is called the adjacent side. It is important to remember that the opposite and adjacent sides are named in relationship to the angle A or the angle we are using in a problem. If the angle had been the top angle, the opposite and adjacent sides would have been switched.

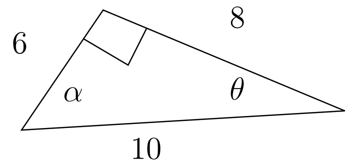
The three trigonometric funtions are functions taken of angles. When an angle goes into the function, the output is a ratio of two of the triangle sides. The ratios are as describe below:

$$\mathbf{sin}\boldsymbol{\theta} = \frac{\textbf{opposite}}{\textbf{hypotenuse}} \qquad \mathbf{cos}\boldsymbol{\theta} = \frac{\textbf{adjacent}}{\textbf{hypotenuse}} \qquad \mathbf{tan}\boldsymbol{\theta} = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

The "weird" variable θ is a greek letter, prounounced "theta" and is close in idea to our letter "t". Often working with triangles, the angles are repesented with Greek letters, in honor of the Ancient Greeks who developed much of Geometry. Some students remember the three ratios by remembering the word "SOH CAH TOA" where each letter if the first word of: "Sine: Opposite over Hypotenuse; Cosine: Adjacent over Hypotenuse; and Tangent: Opposite over Adjacent." Knowing how to use each of these relationships is fundamental to solving problems using trigonometry.

Example 1.

Using the diagram at right, find each of the following: $\sin\theta$, $\cos\theta$, $\tan\theta$, $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$.

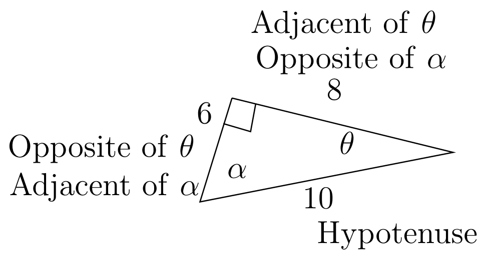


First we will find the three ratios of θ. The hypotenuse is 10, from θ, the opposite side is 6 and the adjacent side is 8. So we fill in the following:

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{6}{8} = \frac{3}{4}$



Now we will find the three ratios of α. The hypotenuse is 10, from α, the opposite side is 8 and the adjacent side is 6. So we fill in the following:

$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$

$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$

$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{6} = \frac{4}{3}$

We can either use a trigonometry table or a calculator to find decimal values for sine, cosine, or tangent of any angle. We only put angle values into the trigonometric functions, never values for sides. Using either a table or a calculator, we can solve the next example.

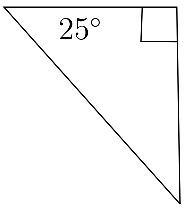
Example 2.

$\sin 42°$	Use calculator or table
0.669	Our Solution
$\tan 12°$	Use calculator or table
0.213	Our Solution
$\cos 18°$	Use calculator or table
0.951	Our Solution

By combining the ratios together with the decimal approximations the calculator or table gives us we can solve for missing sides of a triangle. The trick will be to determien which angle we are working with, naming the sides we are working with, and deciding which trig function can be used with the sides we have.

Example 3.

Find the measure of the missing side.



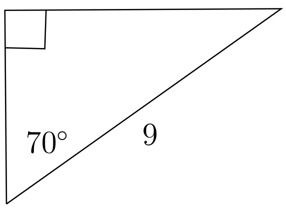
We will be using the angle marked 25°, from this angle, the side marked 4 is the opposite side and the side marked x is the adjacent side.

The trig ratio that uses the opposite and adjacent sides is tangent. So we will take the tangent of our angle.

$\tan 25° = \frac{4}{x}$	Tangent is opposite over adjacent
$\frac{0.466}{1} = \frac{4}{x}$	Evaluate tan25 ∘ , put over 1 so we have proportion
$0.466x = 4$	Find cross product
$\overline{0.466} \quad \overline{0.466}$	Divide both sides by 0.466
$x = 8.58$	Our Solution

Example 4.

Find the measure of the missing side.



We will be using the angle marked 70°. From this angle, the x is the adjacent side and the 9 is the hypotenuse.

The trig ratio that uses adjacent and hypotenuse is the cosine. So we will take the cosine of our angle.

$\cos 70° = \frac{x}{9}$	Cosine is adjacent over hypotenuse
$\frac{0.342}{1} = \frac{x}{9}$	Evaluate cos70°, put over 1 so we have a proportion
$3.08 = 1x$	Find the cross product.
$3.08 = x$	Our Solution.

Practice - Trigonometry

Find the value of each. Round your answers to the nearest ten-thousandth.

1) $\cos 71°$

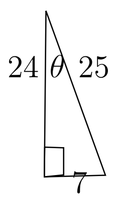
2) $\cos 23°$

3) $\sin 75°$

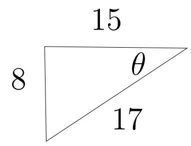
4) $\sin 50°$

Find the value of the trig function indicated.

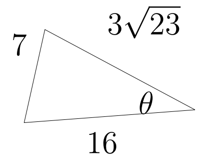
5) $\sin\ \theta$



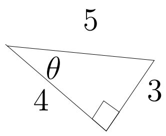
6) $\tan\ \theta$



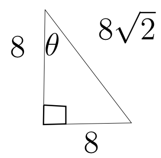
7) $\sin\ \theta$



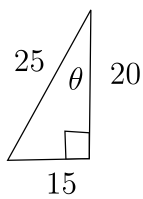
8) $\sin\ \theta$



9) $\sin\ \theta$

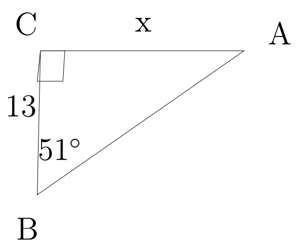


10) $\cos\ \theta$



Find the measure of each side indicated. Round to the nearest tenth.

11)



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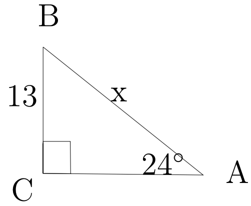

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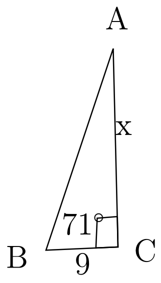

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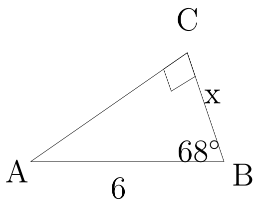

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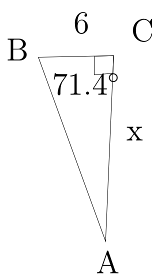

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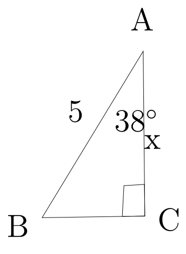

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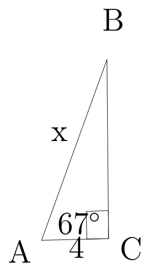

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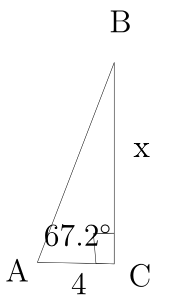

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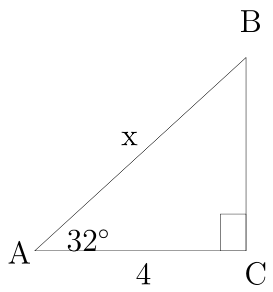

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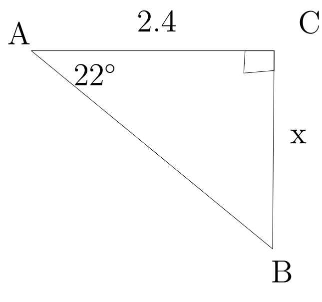

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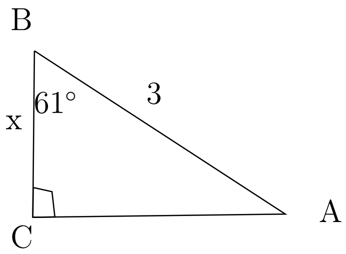

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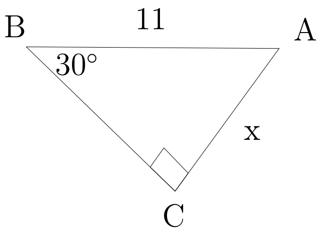

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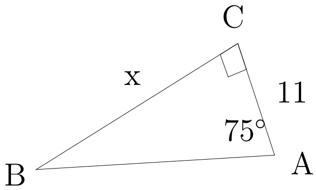

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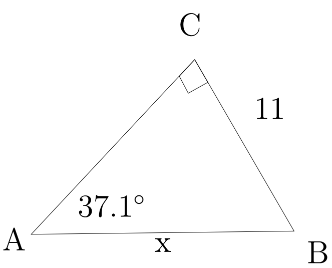

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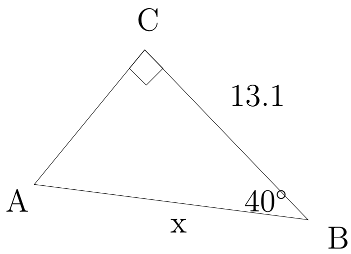

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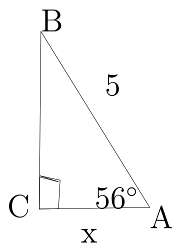

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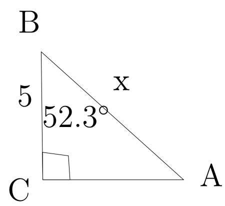

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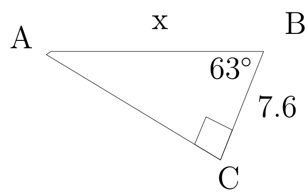

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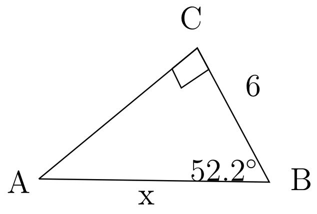

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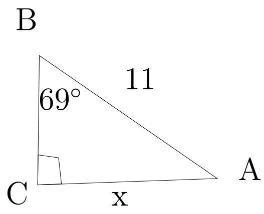

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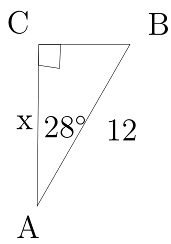

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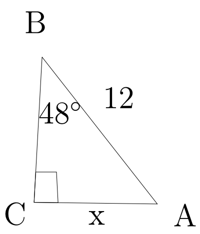

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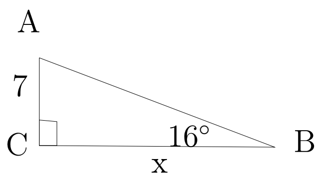

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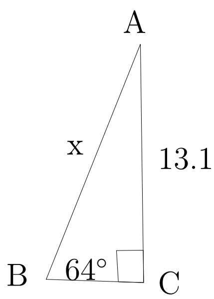

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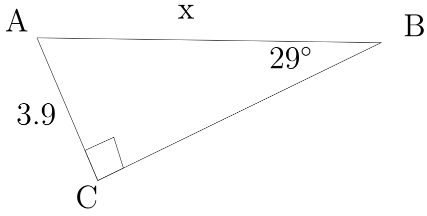

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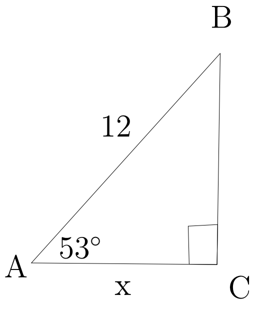

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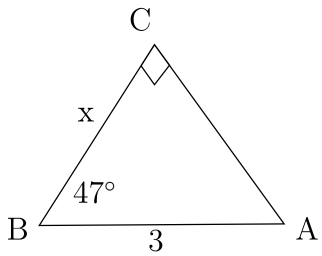

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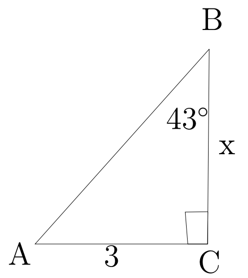

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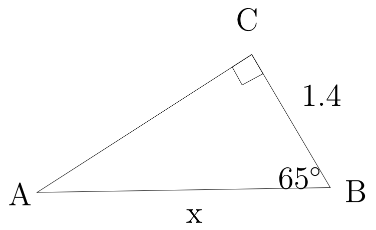

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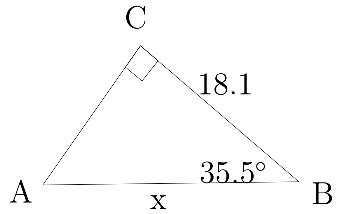

Answers - Trigonometry

1) 0.3256
2) 0.9205
3) 0.9659
4) 0.7660
5) $\frac{7}{25}$
6) $\frac{8}{15}$
7) $\frac{7}{16}$
8) $\frac{3}{5}$
9) $\frac{\sqrt{2}}{2}$
10) $\frac{4}{5}$
11) 16.1
12) 2.8
13) 32
14) 8.2
15) 26.1
16) 16.8
17) 2.2
18) 9.8
19) 17.8
20) 10.3
21) 3.9
22) 10.6
23) 10.2
24) 8.9
25) 9.5
26) 24.4
27) 4.7
28) 14.6
29) 1
30) 8
31) 1.5
32) 7.2
33) 5.5
34)2
35) 41.1
36) 3.2
37) 18.2
38) 3.3
39) 17.1
40) 22.2