

## Solving Linear Equations - Variation

**Objective: Solve variation problems by creating variation equations and finding the variation constant.**

One application of solving linear equations is variation. Often different events are related by what is called the constant of variation. For example, the time it takes to travel a certain distance is related to how fast you are traveling. The faster you travel, the less time it take to get there. This is one type of variation problem, we will look at three types of variation here. Variation problems have two or three variables and a constant in them. The constant, usually noted with a  $k$ , describes the relationship and does not change as the other variables in the problem change. There are two ways to set up a variation problem, the first solves for one of the variables, a second method is to solve for the constant. Here we will use the second method.

The greek letter pi ( $\pi$ ) is used to represent the ratio of the circumference of a circle to its diameter.

**World View Note:** In the 5th centure, Chinese mathematician Zu Chongzhi calculated the value of  $\pi$  to seven decimal places (3.1415926). This was the most accurate value of  $\pi$  for the next 1000 years!

If you take any circle and divide the circumference of the circle by the diameter you will always get the same value, about 3.14159... If you have a bigger circumference you will also have a bigger diameter. This relationship is called **direct variation** or **directly proportional**. If we see this phrase in the problem we know to divide to find the constant of variation.

### Example 1.

$m$  is varies directly as  $n$     "Directly" tells us to divide  

$$\frac{m}{n} = k$$
    Our formula for the relationship

In kickboxing, one will find that the longer the board, the easier it is to break. If you multiply the force required to break a board by the length of the board you will also get a constant. Here, we are multiplying the variables, which means as one variable increases, the other variable decreases. This relationship is called **indirect variation** or **inversly proportional**. If we see this phrase in the problem we know to multiply to find the constant of variation.

### Example 2.

$y$  is inversely proportional to  $z$     "Inversely" tells us to multiply  

$$yz = k$$
    Our formula for the relationship

The formula for the area of a triangle has three variables in it. If we divide the area by the base times the height we will also get a constant,  $\frac{1}{2}$ . This relationship is called **joint variation** or **jointly proportional**. If we see this phrase in the problem we know to divide the first variable by the product of the other two to find the constant of variation.

**Example 3.**

$A$  varies jointly as  $x$  and  $y$  "Jointly" tells us to divide by the product

$$\frac{A}{xy} = k \quad \text{Our formula for the relationship}$$

Once we have our formula for the relationship in a variation problem, we use given or known information to calculate the constant of variation. This is shown for each type of variation in the next three examples.

**Example 4.**

$w$  is directly proportional to  $y$  and  $w = 50$  when  $y = 5$

$$\frac{w}{y} = k \quad \text{"directly" tells us to divide}$$

$$\frac{(50)}{(5)} = k \quad \text{Substitute known values}$$

$$10 = k \quad \text{Evaluate to find our constant}$$

**Example 5.**

$c$  varies indirectly as  $d$  and  $c = 4.5$  when  $d = 6$

$$cd = k \quad \text{"indirectly" tells us to multiply}$$

$$(4.5)(6) = k \quad \text{Substitute known values}$$

$$27 = k \quad \text{Evaluate to find our constant}$$

**Example 6.**

$x$  is jointly proportional to  $y$  and  $z$  and  $x = 48$  when  $y = 2$  and  $z = 4$

$$\frac{x}{yz} = k \quad \text{"Jointly" tells us to divide by the product}$$

$$\frac{(48)}{(2)(4)} = k \quad \text{Substitute known values}$$

$$6 = k \quad \text{Evaluate to find our constant}$$

Once we have found the constant of variation we can use it to find other combinations in the same relationship. Each of these problems we solve will have three important steps, none of which should be skipped.

1. Find the formula for the relationship using the type of variation
2. Find the constant of variation using known values
3. Answer the question using the constant of variation

The next three examples show how this process is worked out for each type of variation.

**Example 7.**

The price of an item varies directly with the sales tax. If a \$25 item has a sales tax of \$2, what will the tax be on a \$40 item?

$$\begin{aligned} \frac{p}{t} &= k && \text{"Directly" tells us to divide price } (p) \text{ and tax } (t) \\ \frac{(25)}{(2)} &= k && \text{Substitute known values for price and tax} \\ 12.5 &= k && \text{Evaluate to find our constant} \\ \frac{40}{t} &= 12.5 && \text{Using our constant, substitute 40 for price to find the tax} \\ \frac{(t)40}{t} &= 12.5(t) && \text{Multiply by LCD } = t \text{ to clear fraction} \\ 40 &= 12.5t && \text{Reduce the } t \text{ with the denominator} \\ \underline{12.5} \quad \underline{12.5} &&& \text{Divide by 12.5} \\ 3.2 = t &&& \text{Our solution: Tax is \$3.20} \end{aligned}$$

**Example 8.**

The speed (or rate) Josiah travels to work is inversely proportional to time it takes to get there. If he travels 35 miles per hour it will take him 2.5 hours to get to work. How long will it take him if he travels 55 miles per hour?

$$\begin{aligned} rt &= k && \text{"Inversely" tells us to multiply the rate and time} \\ (35)(2.5) &= k && \text{Substitute known values for rate and time} \\ 87.5 &= k && \text{Evaluate to find our constant} \\ 55t &= 87.5 && \text{Using our constant, substitute 55 for rate to find the time} \\ \underline{55} \quad \underline{55} &&& \text{Divide both sides by 55} \\ t \approx 1.59 &&& \text{Our solution: It takes him 1.59 hours to get to work} \end{aligned}$$

**Example 9.**

The amount of simple interest earned on an investment varies jointly as the principle (amount invested) and the time it is invested. In an account, \$150 invested

for 2 years earned \$12 in interest. How much interest would be earned on a \$220 investment for 3 years?

$$\begin{aligned} \frac{I}{Pt} &= k && \text{"Jointly" divide Interest (I) by product of Principle (P) \& time (t)} \\ \frac{(12)}{(150)(2)} &= k && \text{Substitute known values for Interest, Principle and time} \\ 0.04 &= k && \text{Evaluate to find our constant} \\ \frac{I}{(220)(3)} &= 0.04 && \text{Using constant, substitute 220 for principle and 3 for time} \\ \frac{I}{660} &= 0.04 && \text{Evaluate denominator} \\ \frac{(660)I}{660} &= 0.04(660) && \text{Multiply by 660 to isolate the variable} \\ I &= 26.4 && \text{Our Solution: The investment earned \$26.40 in interest} \end{aligned}$$

Sometimes a variation problem will ask us to do something to a variable as we set up the formula for the relationship. For example,  $\pi$  can be thought of as the ratio of the area and the radius squared. This is still direct variation, we say the area varies directly as the radius square and thus our variable is squared in our formula. This is shown in the next example.

### Example 10.

The area of a circle is directly proportional to the square of the radius. A circle with a radius of 10 has an area of 314. What will the area be on a circle of radius 4?

$$\begin{aligned} \frac{A}{r^2} &= k && \text{"Direct" tells us to divide, be sure we use } r^2 \text{ for the denominator} \\ \frac{(314)}{(10)^2} &= k && \text{Substitute known values into our formula} \\ \frac{(314)}{100} &= k && \text{Exponents first} \\ 3.14 &= k && \text{Divide to find our constant} \\ \frac{A}{(4)^2} &= 3.14 && \text{Using the constant, use 4 for } r, \text{ don't forget the squared!} \\ \frac{A}{16} &= 3.14 && \text{Evaluate the exponent} \\ \frac{(16)A}{16} &= 3.14(16) && \text{Multiply both sides by 16} \\ A &= 50.24 && \text{Our Solution: Area is 50.24} \end{aligned}$$

When solving variation problems it is important to take the time to clearly state the variation formula, find the constant, and solve the final equation.



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## 1.7 Practice - Variation

**Write the formula that expresses the relationship described**

1.  $c$  varies directly as  $a$
2.  $x$  is jointly proportional to  $y$  and  $z$
3.  $w$  varies inversely as  $x$
4.  $r$  varies directly as the square of  $s$
5.  $f$  varies jointly as  $x$  and  $y$
6.  $j$  is inversely proportional to the cube of  $m$
7.  $h$  is directly proportional to  $b$
8.  $x$  is jointly proportional with the square of  $a$  and the square root of  $b$
9.  $a$  is inversely proportional to  $b$

**Find the constant of variation and write the formula to express the relationship using that constant**

10.  $a$  varies directly as  $b$  and  $a = 15$  when  $b = 5$
11.  $p$  is jointly proportional to  $q$  and  $r$  and  $p = 12$  when  $q = 8$  and  $r = 3$
12.  $c$  varies inversely as  $d$  and  $c = 7$  when  $d = 4$
13.  $t$  varies directly as the square of  $u$  and  $t = 6$  when  $u = 3$
14.  $e$  varies jointly as  $f$  and  $g$  and  $e = 24$  when  $f = 3$  and  $g = 2$
15.  $w$  is inversely proportional to the cube of  $x$  and  $w$  is 54 when  $x = 3$
16.  $h$  is directly proportional to  $j$  and  $h = 12$  when  $j = 8$
17.  $a$  is jointly proportional with the square of  $x$  and the square root of  $y$  and  $a = 25$  when  $x = 5$  and  $y = 9$
18.  $m$  is inversely proportional to  $n$  and  $m = 1.8$  when  $n = 2.1$

**Solve each of the following variation problems by setting up a formula to express the relationship, finding the constant, and then answering the question.**

19. The electrical current, in amperes, in a circuit varies directly as the voltage. When 15 volts are applied, the current is 5 amperes. What is the current when 18 volts are applied?
20. The current in an electrical conductor varies inversely as the resistance of the conductor. If the current is 12 ampere when the resistance is 240 ohms, what is the current when the resistance is 540 ohms?
21. Hooke's law states that the distance that a spring is stretched by hanging object varies directly as the mass of the object. If the distance is 20 cm when the mass is 3 kg, what is the distance when the mass is 5 kg?
22. The volume of a gas varies inversely as the pressure upon it. The volume of a gas is  $200 \text{ cm}^3$  under a pressure of  $32 \text{ kg/cm}^2$ . What will be its volume under a pressure of  $40 \text{ kg/cm}^2$ ?
23. The number of aluminum cans used each year varies directly as the number of people using the cans. If 250 people use 60,000 cans in one year, how many cans are used each year in Dallas, which has a population of 1,008,000?
24. The time required to do a job varies inversely as the number of people working. It takes 5hr for 7 bricklayers to build a park well. How long will it take 10 bricklayers to complete the job?
25. According to Fidelity Investment Vision Magazine, the average weekly allowance of children varies directly as their grade level. In a recent year, the average allowance of a 9th-grade student was 9.66 dollars per week. What was the average weekly allowance of a 4th-grade student?
26. The wavelength of a radio wave varies inversely as its frequency. A wave with a frequency of 1200 kilohertz has a length of 300 meters. What is the length of a wave with a frequency of 800 kilohertz?
27. The number of kilograms of water in a human body varies directly as the mass of the body. A 96-kg person contains 64 kg of water. How many kilograms of water are in a 60-kg person?
28. The time required to drive a fixed distance varies inversely as the speed. It takes 5 hr at a speed of 80 km/h to drive a fixed distance. How long will it take to drive the same distance at a speed of 70 km/h?
29. The weight of an object on Mars varies directly as its weight on Earth. A person weighs 95lb on Earth weighs 38 lb on Mars. How much would a 100-lb person weigh on Mars?

30. At a constant temperature, the volume of a gas varies inversely as the pressure. If the pressure of a certain gas is 40 newtons per square meter when the volume is 600 cubic meters what will the pressure be when the volume is reduced by 240 cubic meters?
31. The time required to empty a tank varies inversely as the rate of pumping. If a pump can empty a tank in 45 min at the rate of 600 kL/min, how long will it take the pump to empty the same tank at the rate of 1000 kL/min?
32. The weight of an object varies inversely as the square of the distance from the center of the earth. At sea level (6400 km from the center of the earth), an astronaut weighs 100 lb. How far above the earth must the astronaut be in order to weigh 64 lb?
33. The stopping distance of a car after the brakes have been applied varies directly as the square of the speed  $r$ . If a car, traveling 60 mph can stop in 200 ft, how fast can a car go and still stop in 72 ft?
34. The drag force on a boat varies jointly as the wetted surface area and the square of the velocity of a boat. If a boat going 6.5 mph experiences a drag force of 86 N when the wetted surface area is  $41.2 \text{ ft}^2$ , how fast must a boat with  $28.5 \text{ ft}^2$  of wetted surface area go in order to experience a drag force of 94N?
35. The intensity of a light from a light bulb varies inversely as the square of the distance from the bulb. Suppose intensity is  $90 \text{ W/m}^2$  (watts per square meter) when the distance is 5 m. How much further would it be to a point where the intensity is  $40 \text{ W/m}^2$ ?
36. The volume of a cone varies jointly as its height, and the square of its radius. If a cone with a height of 8 centimeters and a radius of 2 centimeters has a volume of  $33.5 \text{ cm}^3$ , what is the volume of a cone with a height of 6 centimeters and a radius of 4 centimeters?
37. The intensity of a television signal varies inversely as the square of the distance from the transmitter. If the intensity is  $25 \text{ W/m}^2$  at a distance of 2 km, how far from the transmitter are you when the intensity is  $2.56 \text{ W/m}^2$ ?
38. The intensity of illumination falling on a surface from a given source of light is inversely proportional to the square of the distance from the source of light. The unit for measuring the intensity of illumination is usually the foot-candle. If a given source of light gives an illumination of 1 foot-candle at a distance of 10 feet, what would the illumination be from the same source at a distance of 20 feet?



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## Answers - Variation

1)  $\frac{c}{a} = k$

2)  $\frac{x}{yz} = k$

3)  $wx = k$

4)  $\frac{r}{s^2} = k$

5)  $\frac{f}{xy} = k$

6)  $jm^3 = k$

7)  $\frac{h}{b} = k$

8)  $\frac{x}{a\sqrt[3]{b}} = k$

9)  $ab = k$

10)  $\frac{a}{b} = 3$

11)  $\frac{P}{rq} = 0.5$

12)  $cd = 28$

13)  $\frac{t}{u^2} = 0.67$

14)  $\frac{e}{fg} = 4$

15)  $wx^3 = 1458$

16)  $\frac{h}{j} = 1.5$

17)  $\frac{a}{x\sqrt[2]{y}} = 0.33$

18)  $mn = 3.78$

19) 6 k

20) 5.3 k

21) 33.3 cm

22) 160 kg/cm<sup>3</sup>

23) 241,920,000 cans

24) 3.5 hours

25) 4.29 dollars

26) 450 m

27) 40 kg

28) 5.7 hr

29) 40 lb

30) 100 N

31) 27 min

32) 1600 km

33)  $r = 36$

34) 8.2 mph

35) 2.5 m

36)  $V = 100.5 \text{ cm}^3$

37) 6.25 km

38)  $I = 0.25$



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