Variation Problems

Set 1

1. If \( r \) varies inversely as \( s \) with the constant of variation \( k = \frac{1}{2} \), what is \( r \) when \( s = 5 \)?

2. If \( z \) varies jointly as \( u \) and \( v \) with the constant of variation \( k = -2 \), what is \( z \) when \( u = 2 \) and \( v = 3 \)?

3. If \( y \) varies directly as \( x \) and inversely as \( w \) with the constant of variation \( k = 3 \), what is \( y \) when \( x = 4 \) and \( w = 6 \)?

4. If \( r \) varies inversely as \( s \) and \( t \) with the constant of variation \( k = 3 \), what is \( r \) when \( s = 2 \) and \( t = \frac{1}{5} \)?

5. If \( u \) is inversely proportional to \( t \) and \( s \), and \( u = 3 \) when \( t = 2 \) and \( s = 3 \), what is the constant of proportionality?

6. If \( z \) varies directly as \( x \) and inversely as \( y \), and \( z = 4 \) when \( x = 3 \) and \( y = 5 \), what is \( z \) when \( x = 4 \) and \( y = 6 \)?

7. If \( y \) varies directly as \( t \) and inversely as \( u \) and the square of \( v \), and \( y = 14 \) when \( t = 6 \), \( u = 3 \), and \( v = 2 \), what is \( y \) when \( t = 9 \), \( u = 2 \), and \( v = 3 \)?

8. The volume, \( V \), of a cone varies jointly as its height, \( h \), and the square of its radius, \( r \). If a cone with a height of 8 centimeters and a radius of 2 centimeters has a volume of 33.5 cm\(^3\), what is the volume of a cone with a height of 6 centimeters and a radius of 4 centimeters?

9. The weight of an object varies inversely as the square of its distance from the center of the earth. The radius of the earth is 6400 kilometers. If a man weighs 90 kilograms on the earth's surface, what will he weigh 2000 kilometers above the earth? 5000 kilometers above the earth? (First, choose symbols to represent the two variables, weight and distance.)

10. At a constant temperature, the volume of a gas varies inversely as the pressure. If the pressure of a certain gas is 40 newtons per square meter when the volume is 600 cubic meters, what will the pressure be when the volume is reduced to 240 cubic meters?

11. The resistance of a conductor to the flow of electricity is measured in units called ohms. The resistance of wire to the flow of electricity varies directly as the length of the wire and inversely as the square of its diameter. If 30 meters of copper wire with a diameter of 0.65 centimeter has a resistance of 0.0159 ohm, what is the resistance of 300 meters of copper wire having a diameter of 0.4 centimeter?
12. The maximum distance that a car can travel is directly proportional to the number of liters of gasoline in the tank and inversely proportional to the wind speed. If the maximum distance for a car with 76 liters of gasoline given a wind speed of 48 kilometers per hour is 480 kilometers, what is the maximum distance for that car with 38 liters given a 64 km/hr wind?

13. The intensity of illumination falling on a surface from a given source of light is inversely proportional to the square of the distance from the source of light. The unit for measuring the intensity of illumination is usually the footcandle. If a given source of light gives an illumination of 1 foot-candle at a distance of 10 feet, what would the illumination be from the same source at a distance of 20 feet?

14. Refer to problem 13. What happens if the strength of the light source varies? It turns out that for a given distance, the illumination varies directly with the strength of the light source. Hence the intensity of illumination falling on a surface varies directly with the strength of the light source and inversely with the square of the distance from the light source. If a certain type of 100-watt bulb gives an illumination of 4.80 foot-candles at a distance of 5 feet, how much illumination would three lamps of the same type give at a distance of 10 feet?

15. The volume occupied by a gas varies directly as its absolute temperature and inversely as the pressure it is under. At a temperature of 300° and under a pressure of 5.88 newtons per square centimeter, a sample of gas occupies a volume of 40 milliliters. What volume will this sample of gas occupy at a temperature of 360° and under a pressure of 8.82 newtons?

16. The weight that a beam can support varies jointly as the width of the beam and the square of the thickness and inversely as the length. A beam 5 centimeters wide, 10 centimeters deep, and 275 centimeters long will support a load of 300 kilograms. What is the safe load for a beam of the same length and width but a depth of 15 centimeters?

Set 2
Find the variation constant and an equation of variation if y varies directly as x and the following conditions exist.

1. \(y = 28\) when \(x = 7\)
2. \(y = 5\) when \(x = 12\)
3. \(y = 3.4\) when \(x = 2\)
4. \(y = 2\) when \(x = 5\)
5. \(y = 30\) when \(x = 8\)
6. \(y = 1\) when \(x = \frac{1}{3}\)
7. \(y = 0.8\) when \(x = 0.5\)
8. \(y = 0.6\) when \(x = 0.4\)

Solve.

9. The electric current \(I\), in amperes, in a circuit varies directly as the voltage \(V\). When 15 volts are applied, the current is 5 amperes. What is the current when 18 volts are applied?
10. Hooke's law states that the distance $d$ that a spring is stretched by a hanging object varies directly as the mass $m$ of the object. If the distance is 20 cm when the mass is 3 kg, what is the distance when the mass is 5 kg?

11. The number $N$ of aluminum cans used each year varies directly as the number of people using the cans. If 250 people use 60,000 cans in one year, how many cans are used each year in Dallas, which has a population of 1,008,000?

12. According to Fidelity Investments Investment Vision Magazine, the average weekly allowance $A$ of children varies directly as their grade level, $G$. In a recent year, the average allowance of a 9th-grade student was $9.66 per week. What was the average weekly allowance of a 4th-grade student?

13. The number of kilograms $W$ of water in a human body varies directly as the mass of the body. A 96-kg person contains 64 kg of water. How many kilograms of water are in a 60-kg person?

14. The weight $M$ of an object on Mars varies directly as its weight $E$ on Earth. A person who weighs 95 lb on Earth weighs 38 lb on Mars. How much would a 100-lb person weigh on Mars?

15. The average U.S. community of population 12,500 released about 385 tons of lead into the environment in a recent year. How many tons were released nationally? Use 250,000,000 as the U.S. population.

16. The relative aperture, or f-stop, of a 23.5-mm lens is directly proportional to the focal length $F$ of the lens. If a lens with a 150-mm focal length has an f-stop of 6.3, find the f-stop of a 23.5-mm lens with a focal length of 80 mm.

Find the variation constant and an equation of variation in which $y$ varies inversely as $x$, and the following conditions exist.

17. $y = 6$ when $x = 10$  
18. $y = 16$ when $x = 4$

19. $y = 4$ when $x = 3$  
20. $y = 4$ when $x = 9$

21. $y = 12$ when $x = 3$  
22. $y = 9$ when $x = 5$

23. $y = 27$ when $x = \frac{1}{3}$  
24. $y = 81$ when $x = 9$

Solve.

25. The time $t$ required to empty a tank varies inversely as the rate $r$ of pumping. If a pump can empty a tank in 45 min at the rate of 600 kL/min, how long will it take the pump to empty the same tank at the rate of 1000 kL/min?
26. The current I in an electrical conductor varies inversely as the resistance R of the conductor. If the current is \( \frac{1}{2} \) ampere when the resistance is 240 ohms, what is the current when the resistance is 540 ohms?

27. The volume V of a gas varies inversely as the pressure P upon it. The volume of a gas is 200 cm\(^3\) under a pressure of 32 kg/cm\(^2\). What will be its volume under a pressure of 40 kg/cm\(^2\)?

28. The time T required to do a job varies inversely as the number of people P working. It takes 5 hr for 7 bricklayers to build a park wall. How long will it take 10 bricklayers to complete the job?

29. The wavelength W of a radio wave varies inversely as its frequency F. A wave with a frequency of 1200 kilohertz has a length of 300 meters. What is the length of a wave with a frequency of 800 kilohertz?

30. The time t required to drive a fixed distance varies inversely as the speed r. It takes 5 hr at a speed of 80 km/h to drive a fixed distance. How long will it take to drive the same distance at a speed of 70 km/h?

Find an equation of variation in which:

31. y varies directly as the square of x, and y = 6 when x=3.

32. y varies directly as the square of x, and y = 0.15 when x = 0.1.

33. y varies inversely as the square of x, and y = 6 when x=3.

34. y varies inversely as the square of x, and y = 0.15 when x = 0.1.

35. y varies jointly as x and z, and y = 56 when x = 14 and z=8.

36. y varies directly as x and inversely as z, and y = 4 when x = 12 and z = 15.

37. y varies jointly as x and the square of z, and y = 105 when x = 14 and z = 5.

38. y varies jointly as x and z and inversely as w, and y = \( \frac{3}{2} \) when x = 2, z = 3, and w = 4.

39. y varies jointly as w and the square of x and inversely as z, and y = 49 when w = 3, x = 7, and z = 12.

40. y varies directly as x and inversely as w and the square of z, and y = 4.5 when x = 15, w = 5, and z=2.

41. y varies jointly as x and z and inversely as the product of w and p, and y = \( \frac{3}{28} \) when x = 3, z = 10, w = 7,and p = 8.
42. y varies jointly as x and z and inversely as the square of w, and y = 5 when x = 16, z = 3, and w = 5.

Solve.

43. The stopping distance d of a car after the brakes have been applied varies directly as the square of the speed r. If a car traveling 60 mph can stop in 200 ft, how fast can a car go and still stop in 72 ft?

44. The volume V a given mass of a gas varies directly as the temperature T and inversely as the pressure P. If V = 231 cm³ when T = 42° and P = 20 kg/cm what is the volume when T = 30° and P = 15 kg/cm²?

45. The intensity I of light from a light bulb varies inversely as the square of the distance d from the bulb. Suppose I is 90 W/m² (watts per square meter) when the distance is 5 m. How much further would it be to a point where the intensity is 40 W/m²?

46. The intensity I of a television signal varies inversely as the square of the distance d from the transmitter. If the intensity is 25 W/m² at a distance of 2 km, how far from the transmitter are you when the intensity is 2.56 W/m²?

47. The weight W of an object varies inversely as the square of the distance d from the center of the earth. At sea level (6400 km from the center of the earth), an astronaut weighs 100 lb. How far above the earth must the astronaut be in order to weigh 64 lb?

48. At a fixed temperature, the resistance R of a wire varies directly as the length l and inversely as the square of its diameter d. If the resistance is 0.1 ohm when the diameter is 1 mm and the length is 50 cm, what is the diameter when the resistance is 1 ohm and the length is 2000 cm?

49. Wind resistance, or atmospheric drag, tends to slow down moving objects. Atmospheric drag varies jointly as an object’s surface area A and velocity v. If a car traveling at a speed of 40 mph with a surface area of 37.8 ft² experiences a drag of 222 N (Newtons), how fast must a car with 51 ft² of surface area travel in order to experience a drag force of 430 N?

50. The drag force F on a boat varies jointly as the wetted surface area A and the square of the velocity of the boat. If a boat going 6.5 mph experiences a drag force of 86 N when the wetted surface area is 41.2 ft², how fast must a boat with 28.5 ft² of wetted surface area go in order to experience a drag force of 94 N?
Answers

Set 1
2. –12 4. \( \frac{15}{2} \) 6. \( \frac{40}{9} \) 8. 100.5 cm
10. 100 newtons/m² 12. 180 kilometers 14. 3.60 foot-candles
16. 675 kilograms

Set 2
1. k = 4; y = 4x 3. k = 1.7; y = 1.7x 5. \( k = \frac{15}{4} \); y = \( \frac{15}{4} x \)
7. k = 1.6; y = 1.6x 9. 6 amperes 11. 241,920,000
13. 40 kg 15. 7,700,000 tons 17. k = 60; y = \( \frac{60}{x} \)
19. k = 12; y = \( \frac{12}{x} \) 21. k = 36; y = \( \frac{36}{x} \) 23. k = 9; y = \( \frac{9}{x} \)
25. 27 min 27. 160 cm³ 29. 450 m
31. y = \( \frac{2}{3} x^2 \) 33. y = \( \frac{54}{x^2} \) 35. y = \( \frac{1}{2} xz \)
37. y = 0.3xz² 39. y = \( \frac{4wx^2}{z} \) 41. y = \( \frac{xz}{5wp} \)
43. 36 mph 45. 2.5 m 47. 1600 km
49. about 57.42 mph